Chapter #2

Single Random Variables And Probability Distributions

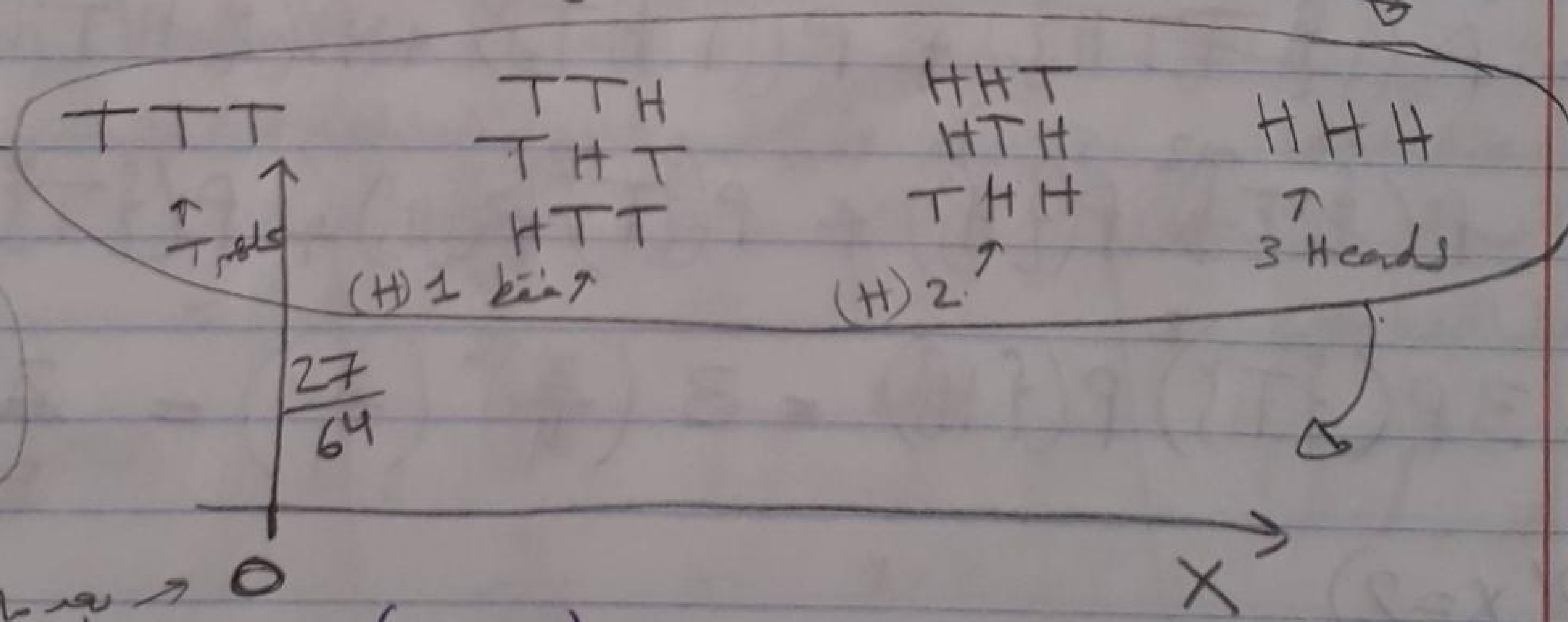
Random variable:- هو عملية mapping من ال Sample space على خط الأعداد.

Ex:- Consider the experiment of flipping a coin for three times. Assume probability of observing a head in a trial is $\frac{1}{4}$.

Let the random variable X, represent the number of heads observed in the three trials.

(a) Determine the sample space.

$$S = \{TTT, TTH, THT, HTT, HHT, THH, HTH, HHH\}.$$



هاد ال Sample space على خط الأعداد، بيشكل تشكيل وتكوين وتعريف ال Random Variable وهو عبارة عن أرقام على خط الأعداد.

بعد ما طلعنا فرع (b)

(b) Compute $P(X=0)$?

$$P(X=0) = P(\{TTT\})$$

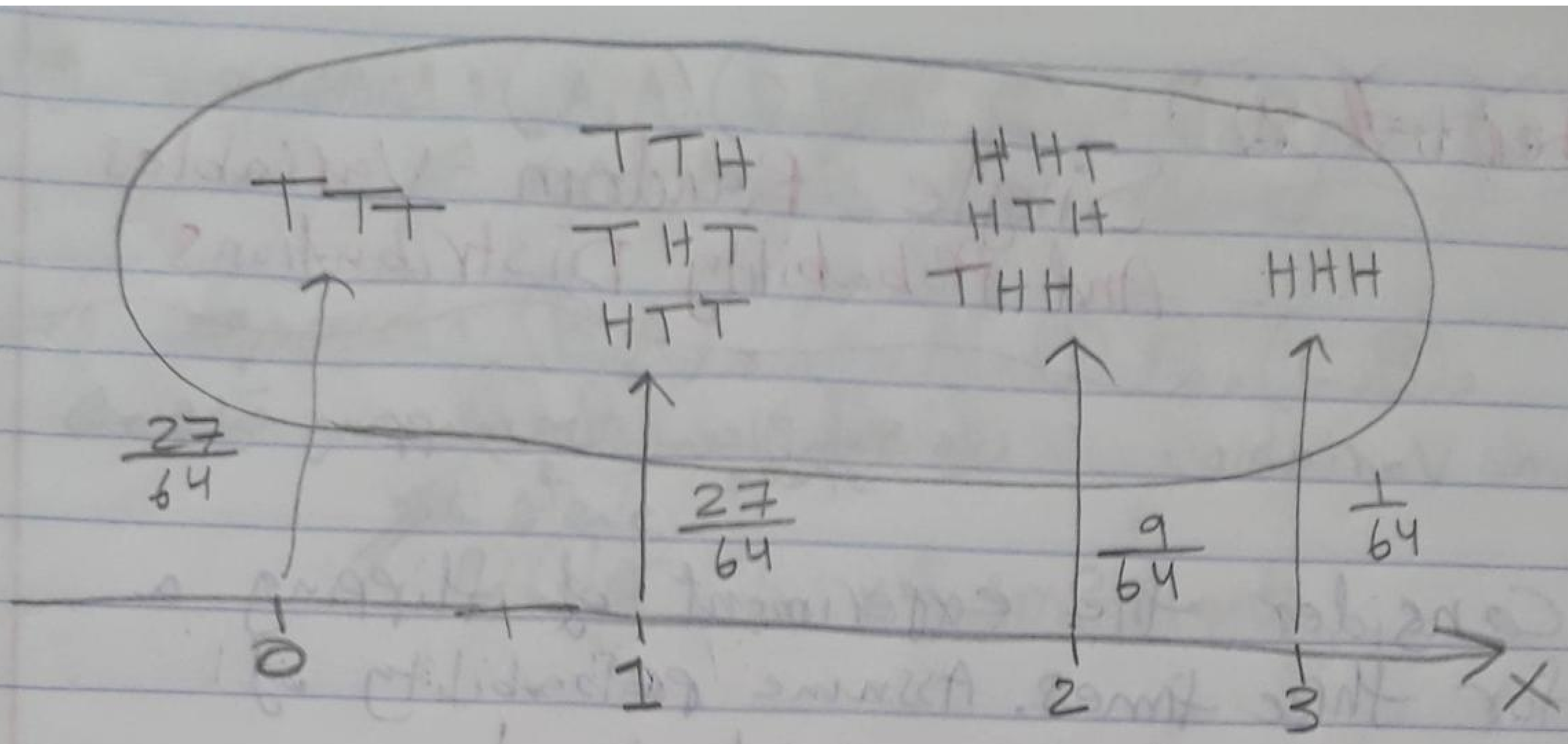
عوضنا هيك، عشان الغرض فوق إنو x = عدد ال heads، فهو بيحكيلي شو الاحتمالية لما عدد ال head=0 ؟ إذاً بمعنى آخر، كل ال tails (TTT)

$$= (P(T))^3$$

$$= [1 - P(H)]^3$$

$$= \left(1 - \frac{1}{4}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

Scanned by TapScanner



(c) $P(X=1)=?$

$P(X=1) = P(\{TTH, THT, HTT\})$

$= P(\{TTH\} \cup \{THT\} \cup \{HTT\})$

disjoint

$= P(\{TTH\}) + P(\{THT\}) + P(\{HTT\})$

$= P(\{T\})^2 P(\{H\}) + P(\{T\})^2 P(\{H\}) + P(\{T\})^2 P(\{H\})$

$= 3P(\{T\})^2 P(\{H\}) = 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{27}{64}$

(d) $P(X=2)$

$P(X=2) = P(\{HHT, HTH, THH\})$

$= P(\{HHT\} \cup \{HTH\} \cup \{THH\})$

$= P(\{HHT\}) + P(\{HTH\}) + P(\{THH\})$

$= 3P(\{H\})^2 P(\{T\})$

$= 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$

$= \frac{9}{64}$

(e) $P(X=3) = ??$

$P(X=3) = P(\{HHH\}) = P(\{H\})^3 = \left(\frac{1}{4}\right)^3$

$= \frac{1}{64}$

(f) $P(X=4) = ??$

$P(X=4) = 0$ → لأنو ما في ولا event بالـsample space موجود فيها 4 heads ، وكذلك الأمر لـ $X=5$ ، أو 3.5 ، أو 3.1 ، أو ... ، أو ...

(g) Determine the probability mass function of the Random Variable X.

اسمها هيك عشان بتعطي كتلة لكل قيمة من قيم X (الـvalue) عن طريق إعطائنا البروبابيليتي .

بترجع البروبابيليتي نفسها كزا كزا ← ... إذا كانت قيمة الـvalue كزا كزا

$$P(X=x) = \begin{cases} \frac{27}{64}, & x=0 \\ \frac{27}{64}, & x=1 \\ \frac{9}{64}, & x=2 \\ \frac{1}{64}, & x=3 \\ 0, & \text{otherwise} \end{cases}$$

otherwise: O.W اختصارها

X → Random Variable دائمًا تُكتب capital letter.

x → value (تُكتب small letter)

Note: PMF is valid if:-

$\boxed{1}$ $P(X=x) \geq 0$

$\boxed{2}$ $\sum_{x=-\infty}^{\infty} P(X=x) = 1$ → حاصل جمع كل الـProbabilities = 1 ، لأنو أساسًا إحنا مسكنا كل event بالـsample space وأعطيناها الـmass يعني Probability ، فحاصل جمعهم كلهم = 1 .

Scanned by TapScanner

كيوميولاتيف تراكمي

Note:- Cumulative Distribution Function (CDF)

$$F_X(x) \triangleq P(X \leq x)$$

by definition

(h) $F_X(-1) = ??$

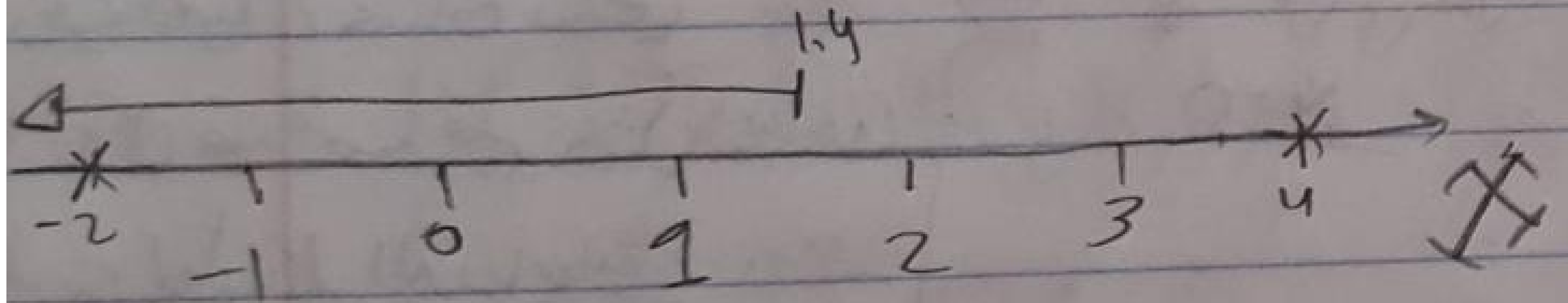
$$F_X(-1) = P(X \leq -1) = 0$$

(i) $F_X(0.5) = ?$

$$F_X(0.5) = P(X \leq 0.5) = P(X=0) = \frac{27}{64}$$

نجمع كل الـ Probability اللي أقل أو تساوي 0.5 (مع حفظ الأعداد طبعاً) فما في Probability إلا عند x=0 .

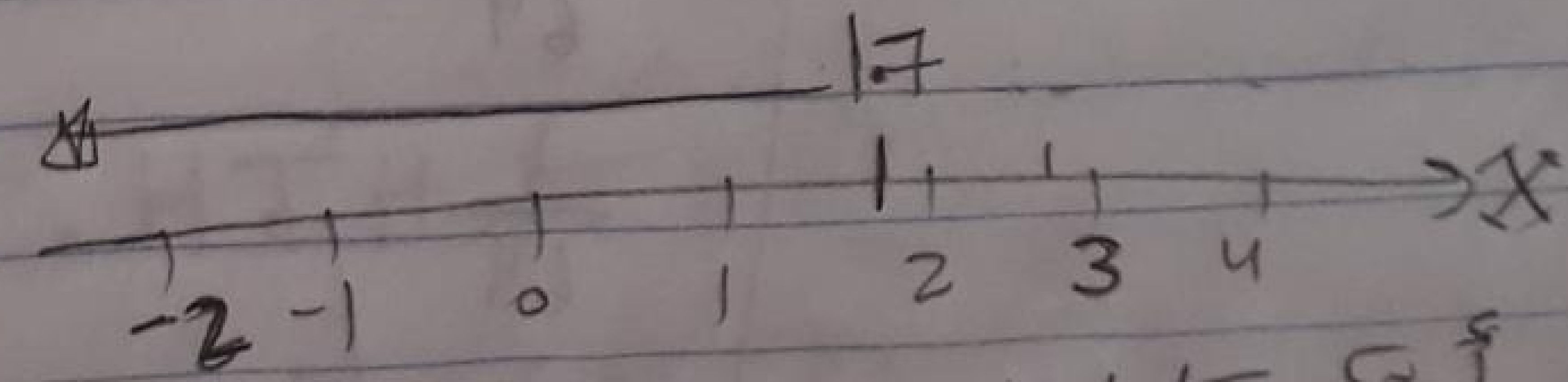
(j) $F_X(1.4) = P(X \leq 1.4) = P(X=0) + P(X=1)$



$$= \frac{27}{64} + \frac{27}{64} = \frac{54}{64}$$

(k) $F_X(1.7) = P(X \leq 1.7) = P(x=0) + P(x=1)$

$$= \frac{54}{64}$$



لاحظ أنه كلما زادت قيمة X ، زادت قيمة الـ CDF ، لأننا نأخذ قيم Probabilities أكبر ، ونجمعهم . وبما أننا نعرف أن الـ Probability ممنوع تكون سالبة ، إذاً هو فقط يزيد أو يبقى كما هو ، زي (j) و (k)

$$F_X(x_1) \leq F_X(x_2) \quad ; \text{ if } \underset{1.4}{x_1} \leq \underset{1.7}{x_2}$$

(L) $F_x(0) = ??$

$F_x(0) = P(X \leq 0) = P(X=0) = \frac{27}{64}$

Zero أكبر من exactly بشوي كمان

(M) $F_x(0^+) = ??$

$F_x(0^+) = P(X \leq 0) = P(X=0) = \frac{27}{64}$

متساويات

$\therefore$ Always: $F_x(0^+) = F_x(0)$

(N) $F_x(0^-) = ?$

$F_x(0^-) = P(X < 0) = 0$

ولا يساويه

(O) $F_x(1)$, $F_x(1^+)$, $F_x(1^-)$?

$F_x(1) = F_x(1^+) = P(X \leq 1) = P(X=0) + P(X=1)$

$= \frac{27}{64} + \frac{27}{64} = \frac{54}{64}$

$F_x(1^-) = P_x(X < 1) = P(X=0) = \frac{27}{64}$

دون مساواة

(P) $F_x(-\infty) = ?$

$F_x(-\infty) = P(X \leq -\infty) = 0$

(9) $F_X(\infty) = ?$

$$F_X(\infty) = P(X \le \infty) = 1$$

عادةً ما بنكتبها 100% إنما 1 ، لأنو هاد ال cdf هو Function وليس Probability
هو عبارة عن مجموعة Probabilities ، فما بزبط أكتبه كنسبة ، ولو
إنو الحكي صحيح ، بس كمفهوم ما بنكتب كنسبة .

(R) Determine and Plot the CDF of X

دائماً المساواة من اليمين

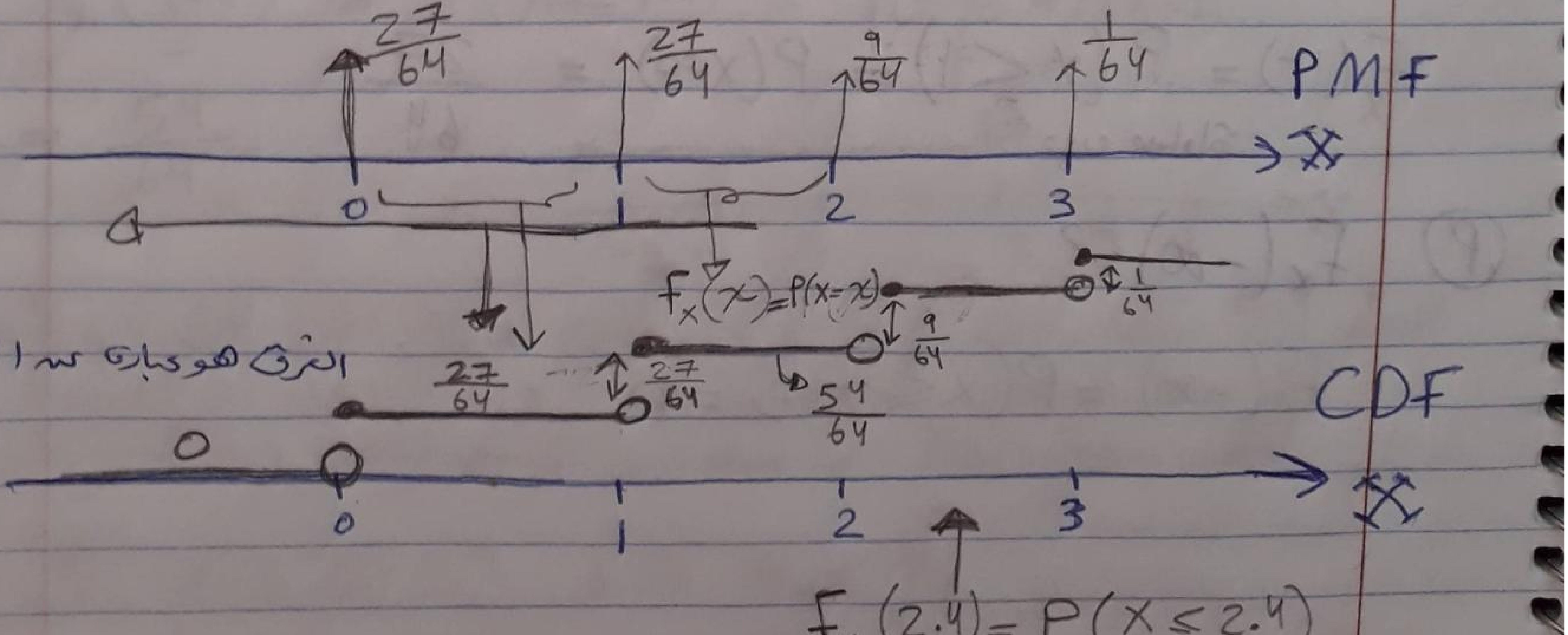
$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{27}{64}, & 0 \le x < 1 \\ \frac{54}{64}, & 1 \le x < 2 \\ \frac{63}{64}, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

$F_X(-3) = 0$

$F_X(-1.23) = 0$

$F_X(0.5) = F_X(0.73) = \frac{27}{64}$

$F_X(1.6) = F_X(1.74) = \frac{54}{64}$



$F_X(2.4) = P(X \le 2.4)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{63}{64}$

* بال PMF دايمًا مجموع القيم اللي عالبيدل لازم يساووا 1 ، لأنهم مجموع probabilities .

* أمّا بال CDF هم أصلًا already مجموعين ، فمجموعهم مش لازم يساوي واحد ، ولازم يحققوا الشروط ، ولازم يكون متصاعد ، ولو ما في otherwise ، لكل القيم رح يكون عنده values ومتصل من اليمين .

Example:- Let Y be a Random Variable with the following CDF

$$F_Y(\underline{y}) = \begin{cases} k, & y < -1 \\ G, & -1 \le y < 3 \\ 0.7, & 3 \le y < 5 \\ H, & 5 \le y \end{cases}$$

$\Rightarrow$ wrong:- ~~$P(-1 \le y < 3)$~~ $= 0.5$

But $F_Y(?) =$ any value between -1 and 3, including -1 equals $\boxed{0.5}$

Assume $P(y=-1) = 0.5$

(a) Assume the values of the constants k, G, and H?

حسب أول property

$F_Y(-\infty) = 0 = k \Rightarrow \boxed{k=0}$

$F_Y(\infty) = 1 \Rightarrow \boxed{H=1}$

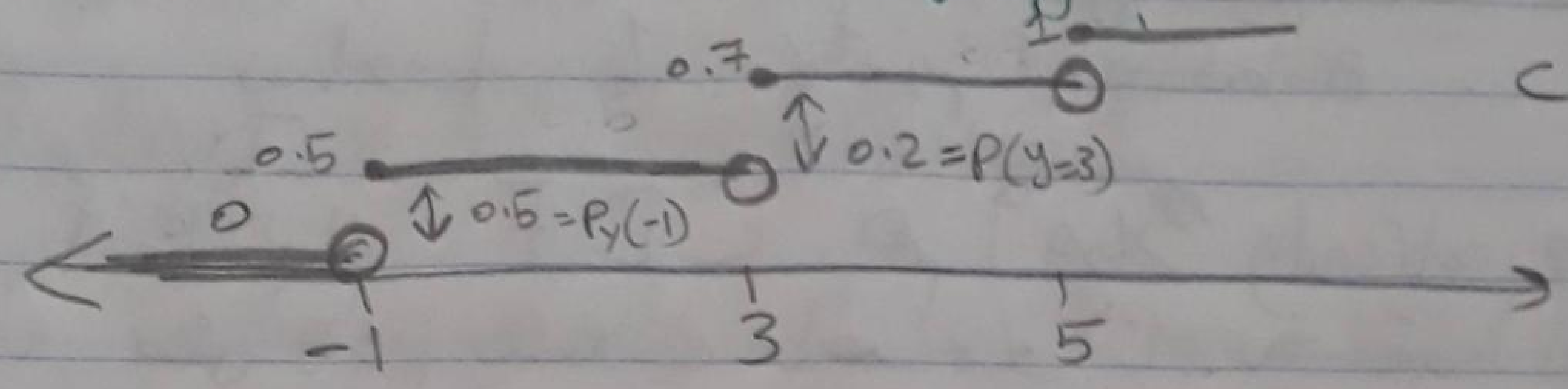
ليش؟ لأنه الـ ∞ تقع في الـ range تبع $5 \le y$

$P(y=-1) = P(y \le -1) - P(y < -1)$

0.5 دي معطى $=$ $G - 0$

$\therefore \boxed{G = 0.5}$

(b) Determine the PMF of y?



$P(y=-1) = 0.5$

$P(y=3) = P(y \leq 3) - P(y<3) = F_y(3) - F_y(3^-)$

$= 0.7 - 0.5$

$= 0.2$

$P(y=5) = P(y \leq 5) - P(y<5)$

$= F_y(5) - F_y(5^-)$

$= 1 - 0.7$

$= 0.3$

لما أخذنا عند النقاط المتصلة زي $y=4$ هو و

$P(y=4) = F_y(4) - F_y(4^-) = 0.7 - 0.7 = 0$ ف الفرق = صفر

إذن بناخذ النقاط الي فيها discontinuity

$$P(Y=y) = \begin{cases} 0.5, & y=-1 \\ 0.2, & y=3 \\ 0.3, & y=5 \\ 0, & \text{O.W} \end{cases}$$ PMF of Y

Example:- Let X be a random Variable with the CDF given as:-

$$F_X(X) = \begin{cases} K & X < -3 \\ 0.2 & -3 \leq X < 0 \\ G & 0 \leq X < 1 \\ H & 1 \leq X \end{cases}$$ CDF

Assume $\underline{P}(X>0) = 0.3$

(a) Determine the values of the constants K, G, and H.

$F_X(-\infty) = 0 \Rightarrow -\infty < -3 < X$

$\therefore \boxed{K=0}$

$F_X(\infty) = 1 \Rightarrow \infty > 1 \geq X$

$\therefore F_X(\infty) = H = 1 \Rightarrow \boxed{H=1}$

We know that:- $P(X>0) + P(X \leq 0) = 1$

لأنه يشمل كل
فضاء الأعداد
(جميع الأرقام التي أكبر من صفر
والتي أصغر من صفر
والتي تساوي الصفر
احتماليتها = 1)

But it is given in the question that:-
$P(X>0) = 0.3$

$\therefore P(X>0) + P(X \leq 0) = 1$

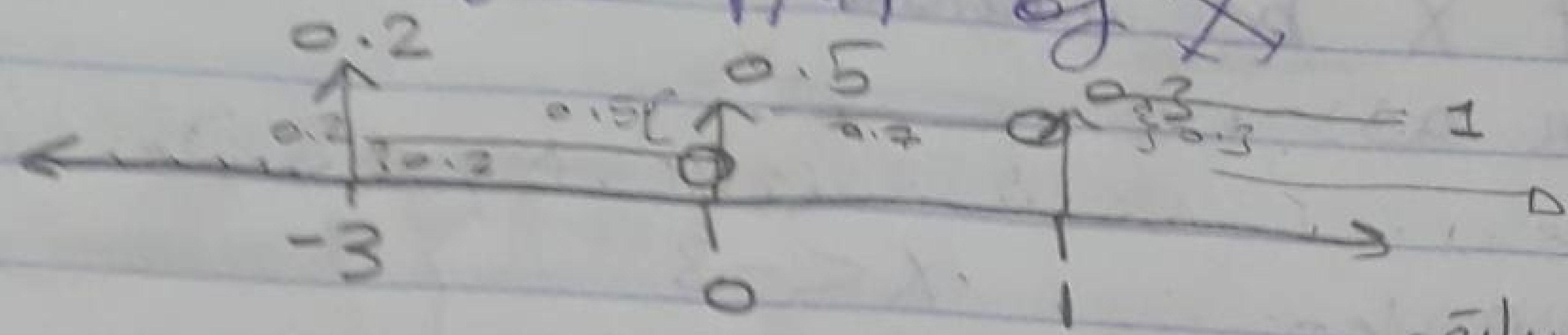
$\therefore P(X \leq 0) = 1 - P(X>0)$

$= 1 - 0.3$

$= 0.7 \Rightarrow F_X(0) = P(X \leq 0) = G$

$\therefore \boxed{G = 0.7}$

(b) Determine the PMF of X



مقدار القفزات
(لازم تاخد بالك) هو البروبابيليتي

$P(-3) = P(X \leq 3) - P(X < 3) = F_x(-3) - F_x(-3^-)$
$= 0.2 - 0$
$= 0.2$

$P(0) = P(X \leq 0) - P(X < 0) = F_x(0) - F_x(-0^-)$
$= 0.7 - 0.2$
$= 0.5$

$P(1) = P(X \leq 1) - P(X < 1) = F_x(1) - F_x(1^-)$
$= 1 - 0.7$
$= 0.3$

$0.W = 0$

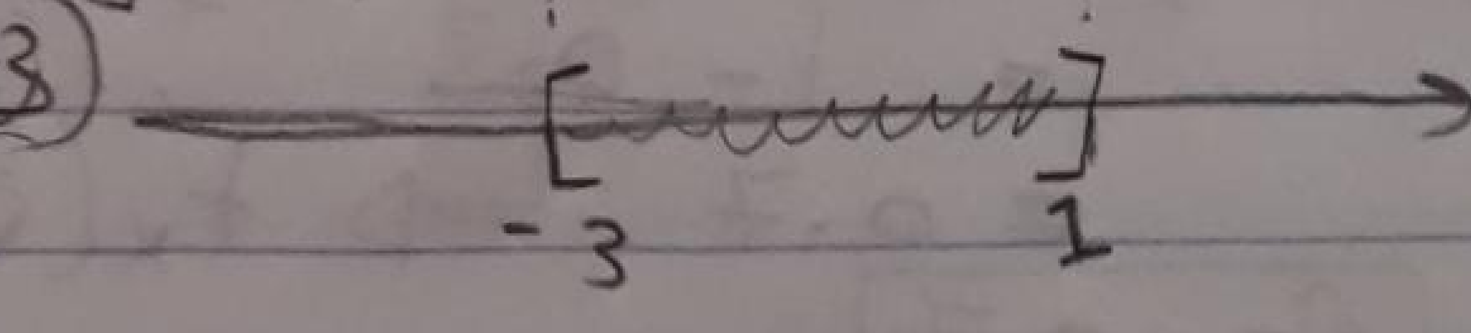
$$P(X=x) \begin{cases} 0.2 & ; \quad x = -3 \\ 0.5 & ; \quad x = 0 \\ 0.3 & ; \quad x = 1 \\ 0 & ; \quad 0.W \end{cases}$$

PMF of R.V X is valid as
$0.2 + 0.5 + 0.3 + 0 = 1$
لازم يطلع واحد .

(c) $P(-3 \leq X \leq 1) = ?$

نفس بعض
ناقص

$P(-3 \leq X \leq 1) = P(X \leq 1) - P(X < -3)$



عكس بعض

$= F_X(-1) - F_x(-3^-) = 1 - 0 = 1$

(d) $P(-3 < x \leq 1) = ??$

$$P(-3 < x \leq 1) = P(x \leq 1) - P(x \leq -3) = F_x(1) - F_x(-3)$$
$$= 1 - 0.2$$
$$= 0.8$$

(e) $P(-3 < x < 1) = ?$

$$P(-3 < x < 1) = P(x < 1) - P(x \leq -3)$$
$$= F_x(1^-) - F_x(-3)$$
$$= 0.7 - 0.2$$
$$= 0.5$$

(f) $P(-3 \leq x < 1) = ??$

$$P(-3 \leq x < 1) = P(x < 1) - P(x < -3)$$
$$= F_x(1^-) - F_x(-3^-)$$
$$= 0.7 - 0$$
$$= 0.7$$

$S = \{0, 1, 2, 3, 4\}$ ← Discrete

$S = \{0, 1, 2, 3, 4, \ldots\ldots\}$ ← Discrete

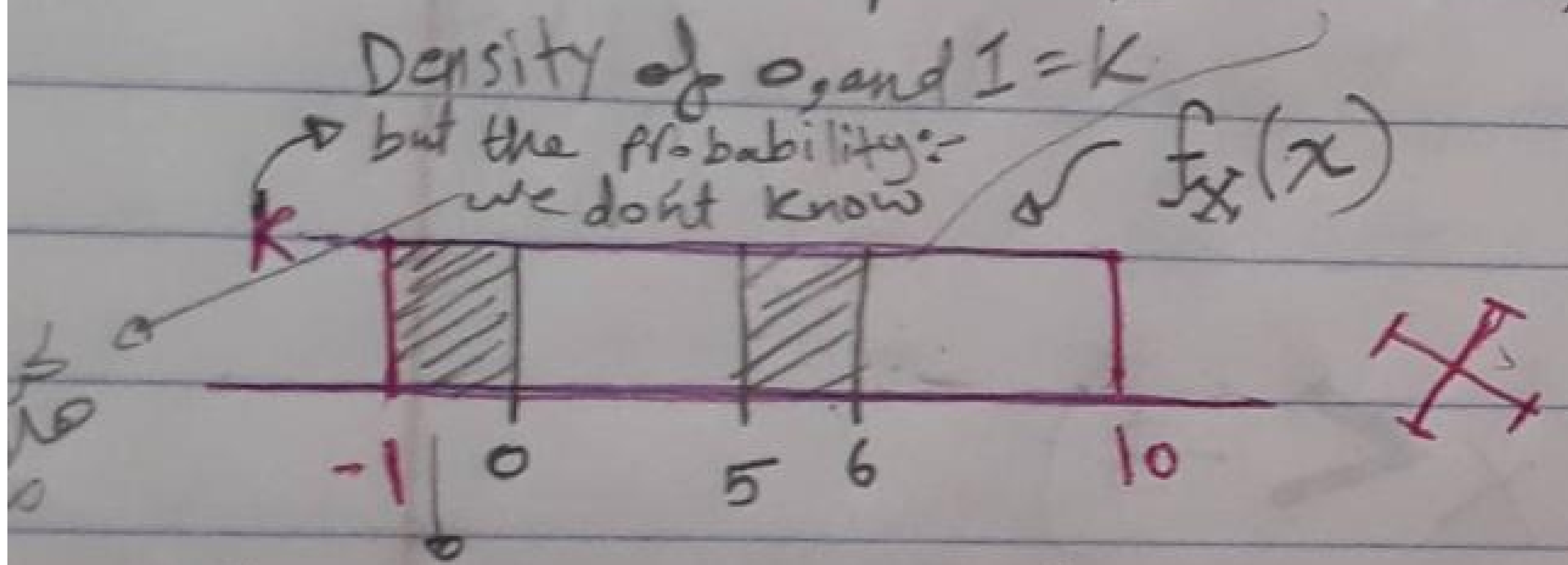
$S = \{-1, -0.5, 0, 0.5, 1, 1.5, \ldots\}$ ← Discrete

$S = \{0 \leq a \leq 2\}$ ← Continuous

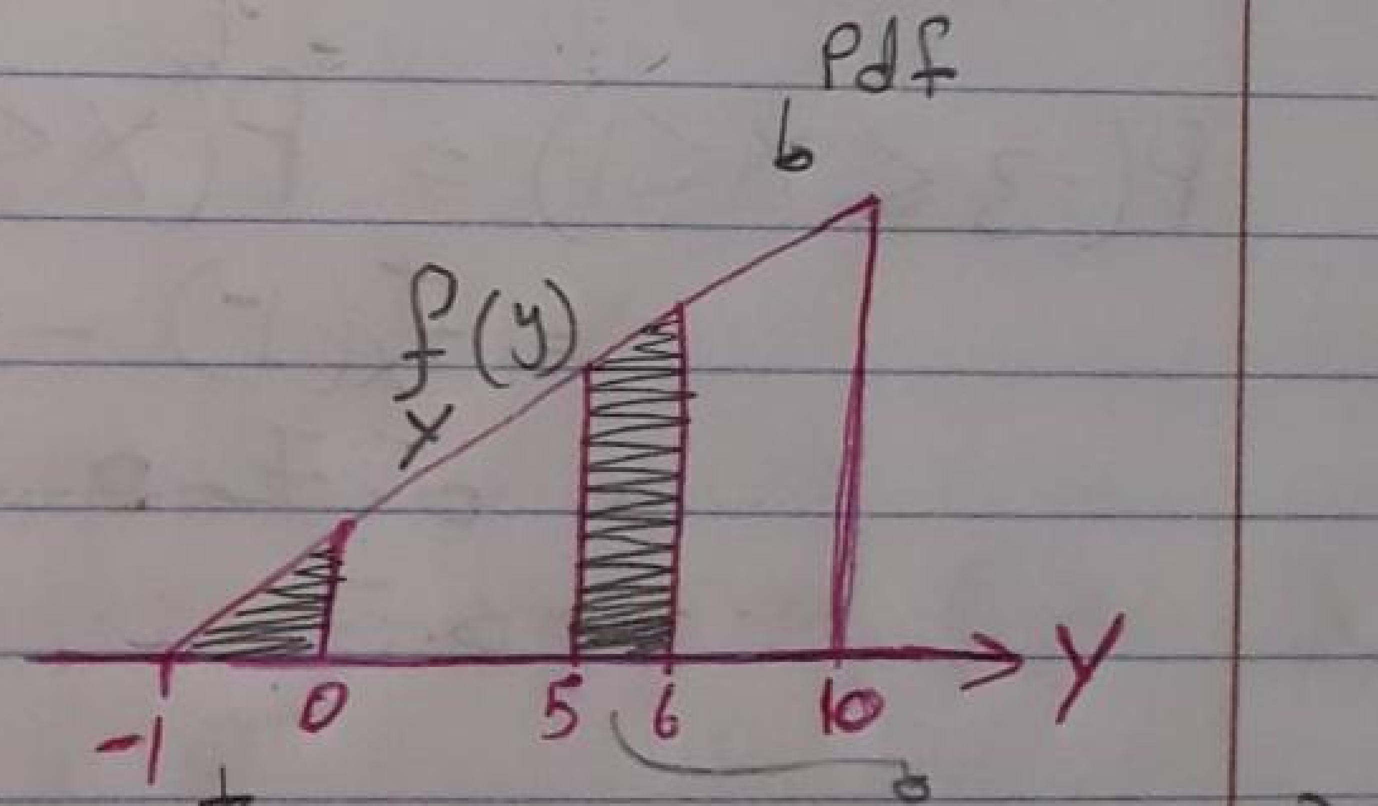
$\in$ Real Numbers

$S = \{-1 \leq b\}$ ← Continuous.

Probability Density Function (Pdf)



$P(-1 \leq X \leq 0) = P(5 \leq X \leq 6$



$P(-1 \leq Y \leq 0) < P(5 \leq Y \leq 6)$

كما الارتفاع اختلف

Properties of $f_X(x)$:-

1. $f_X(x) \geq 0$: nonnegative.

2. $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

3. $P\{x_1 \leq X \leq x_2\} = \int_{x_1}^{x_2} f_X(u)\,du$; Probability is the area under the $f_X(x)$ curve between x_1 and x_2.

Scanned by TapScanner

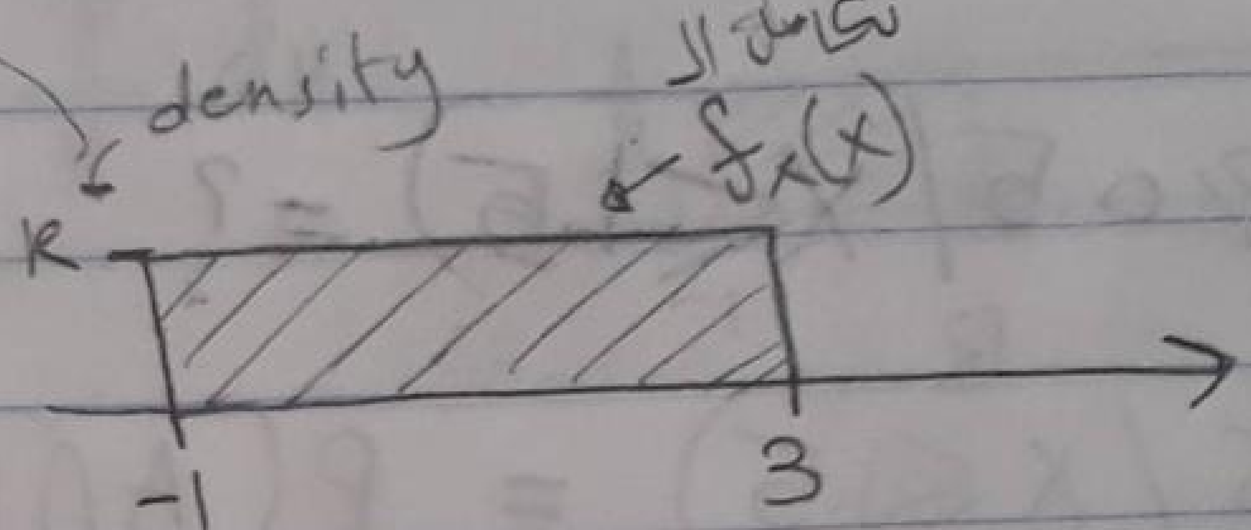
ex:- let X be a Random Variable with the following Pdf :-

$$f_X(x) = \begin{cases} K & , \ -1 \leq x \leq 3 \\ 0 & \text{o.w} \end{cases}$$

ⓐ Determine the Value of the constant K.

K must be (+) → positive

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$



$$= \int_{-\infty}^{-1} \cancel{0}\, dx + \int_{-1}^{3} k\, dx + \int_{3}^{\infty} \cancel{0}\, dx$$

$$= kx \Big|_{-1}^{3} \Rightarrow \boxed{4k = 1} \Rightarrow \therefore \boxed{k = \frac{1}{4}} = \boxed{0.25}$$

حل آخر :-

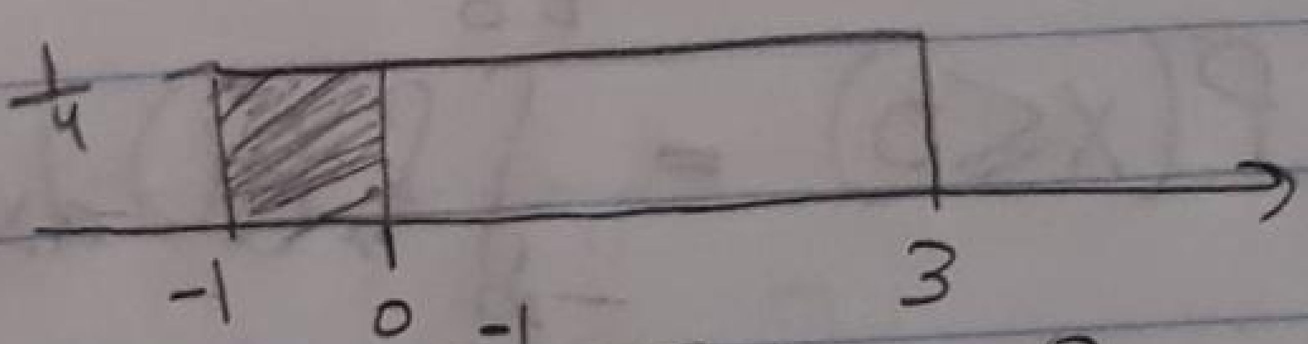
المساحة تحت المنحنى $= 1 =$ Pdf

∴ الشكل مستطيل = الطول × العرض

$K \times 4 = 1$

$\therefore \boxed{K = \frac{1}{4}}$

ⓑ $P(X \leq 0) = ?$



$$P(X \leq 0) = \int_{-\infty}^{0} f_X(x)\, dx = \int_{-\infty}^{-1} \cancel{0}\, dx + \int_{-1}^{0} \frac{1}{4}\, dx$$

$$= \frac{1}{4}(0+1) = \frac{1}{4}$$ → وهي نسبة المساحة المظللة من المساحة الكلية

ما أخذنا مساواة بعين الاعتبار عند الكل عشان الفنكشن .cont بالتالي فقط قيمة مساحة الصفر ستكون الصفر

(c) $P(0 < X \leq 2.5) = ?$

الحد الأعلى

$$P(0 < X \leq 2.5) = \int_0^{2.5} f_x(x)\,.dx = \int_0^{2.5} \frac{1}{4}\,dx$$

الحد الأدنى

$$= \frac{2.5}{4} = \frac{5}{8} = 0.625$$

(d) $P(X \geq 0.5 / X \leq 1.5) = ?$

$$P(\underbrace{X \geq 0.5}_{A} / \underbrace{X \leq 1.5}_{B}) = \frac{P(A \cap B)}{P(B)} = \frac{P(X \geq 0.5 \cap X \leq 1.5)}{P(X \leq 1.5)}$$

$$= \frac{P(0.5 \leq X \leq 1.5)}{P(X \leq 1.5)} = \frac{\text{تكامل البسط}}{\text{تكامل المقام}}$$

$$= \frac{\int_{0.5}^{1.5} \frac{1}{4}\,dx}{\int_{-1}^{1.5} \frac{1}{4}\,dx} = \frac{\cancel{\frac{1}{4}}[1.5 - 0.5]}{\cancel{\frac{1}{4}}[1.5 - -1]} = \frac{1}{2.5} = 0.4$$

(e) $F_x(0) = ?$

cdf

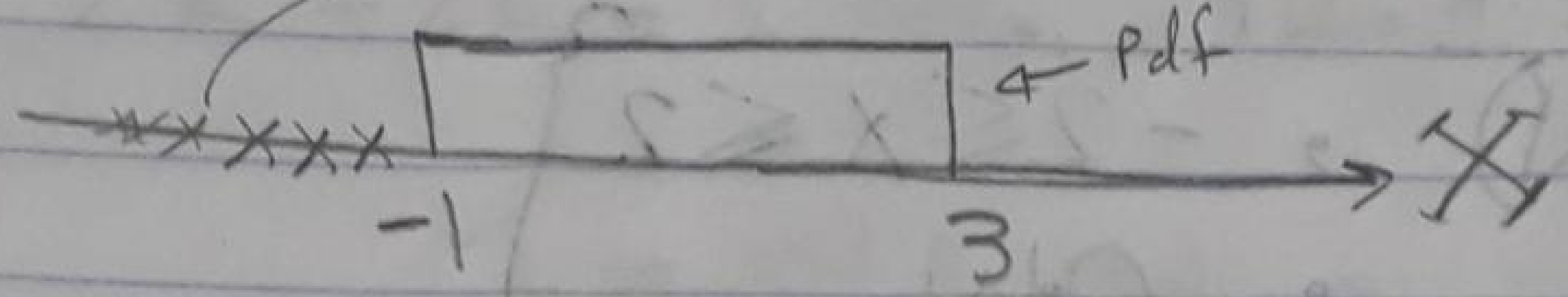
$$F_x(0) = P(X \leq 0) = \int_{-\infty}^{0} f_x(x)\,dx = \frac{1}{4}$$

(F) $F_x(x) = ?$

$$F_x(x) = \int_{-\infty}^{x} f_x(u)\,du$$

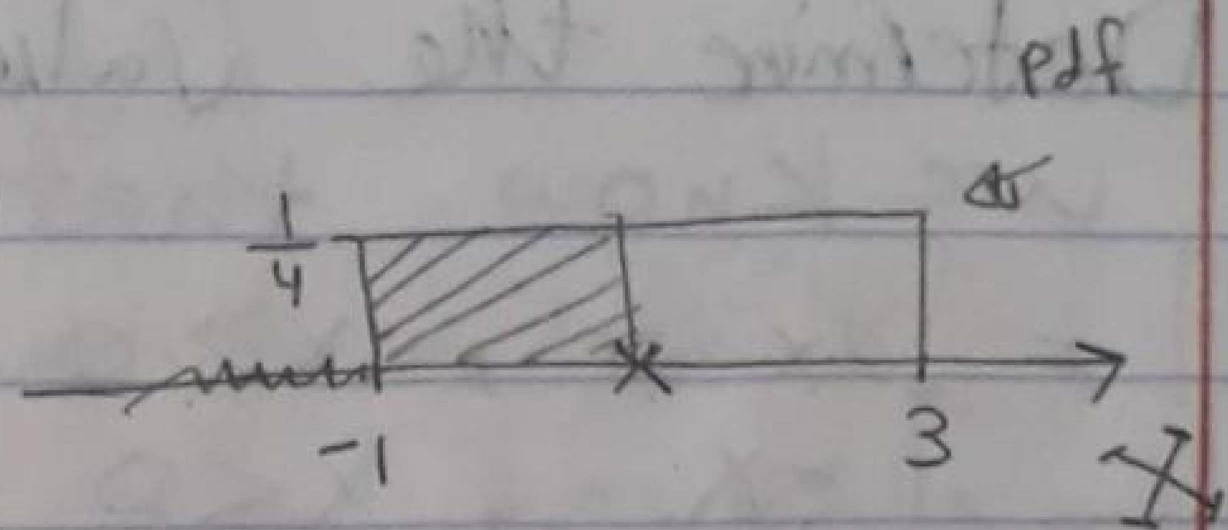
تكملة

I) $x < -1 \Rightarrow F_X(x) = \int_{-\infty}^{x} 0 \, dx = 0$



II) $-1 < x < 3$

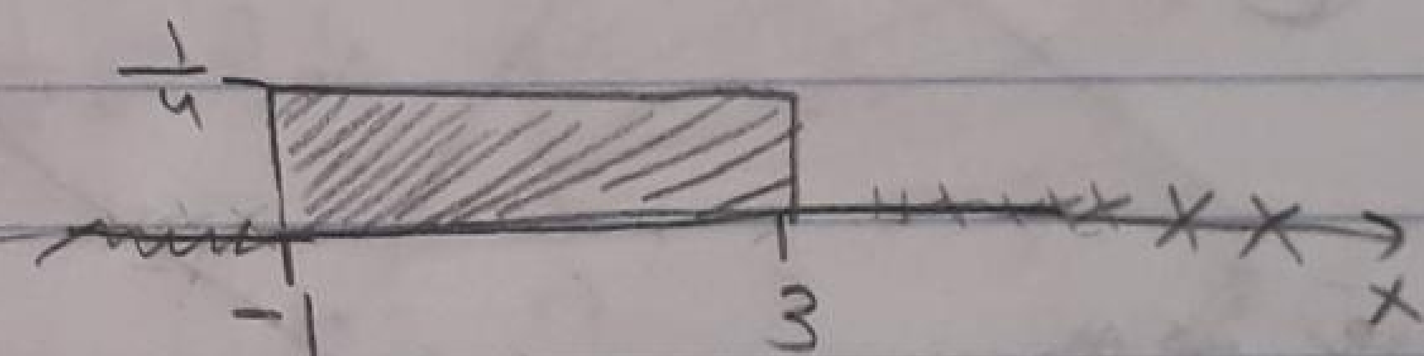
$$F_X(x) = \int_{-\infty}^{-1} \cancel{0} \, dx + \int_{-1}^{x} \frac{1}{4} \, du$$



$$= \frac{1}{4}[x + 1]$$

$$= \frac{1}{4}[x+1]$$

III) $3 < x$

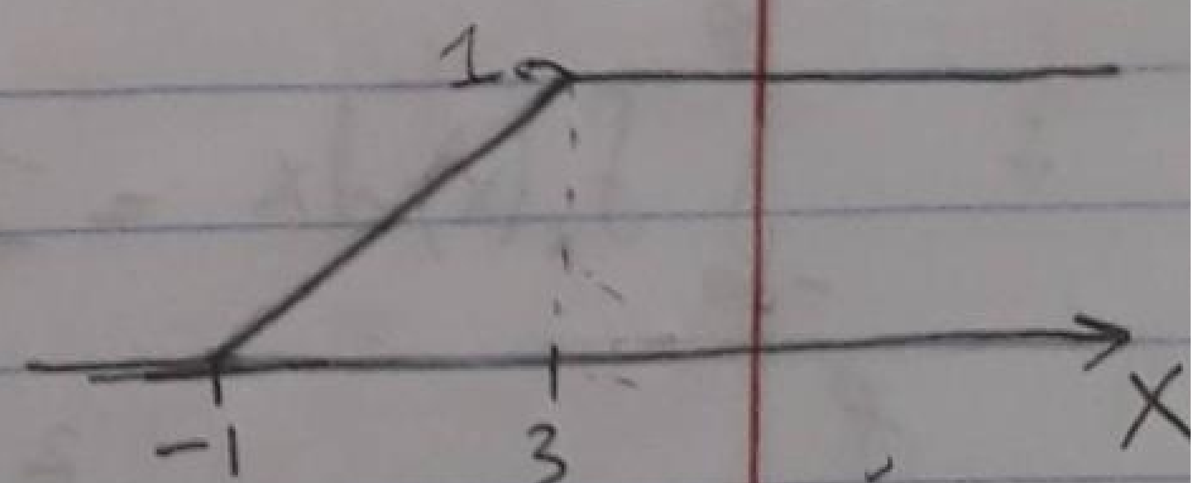


$$\therefore F_X(x) = \int_{-\infty}^{-1} \cancel{0} \, du + \underbrace{\int_{-1}^{3} \frac{1}{4} \, du}_{1} + \int_{3}^{x} \cancel{0} \, du$$

$$= 1$$

$$\therefore F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{4}[x+1] & -1 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

cdf of X



المساواة دائماً على يمين الفترة، عشان هيك هيك وهنا حطينا المساواة.

* الفنكشن دائماً continuous لما يكون في pdf.

Example :- Let X be a Random Variable (R.V) with the following Pdf →

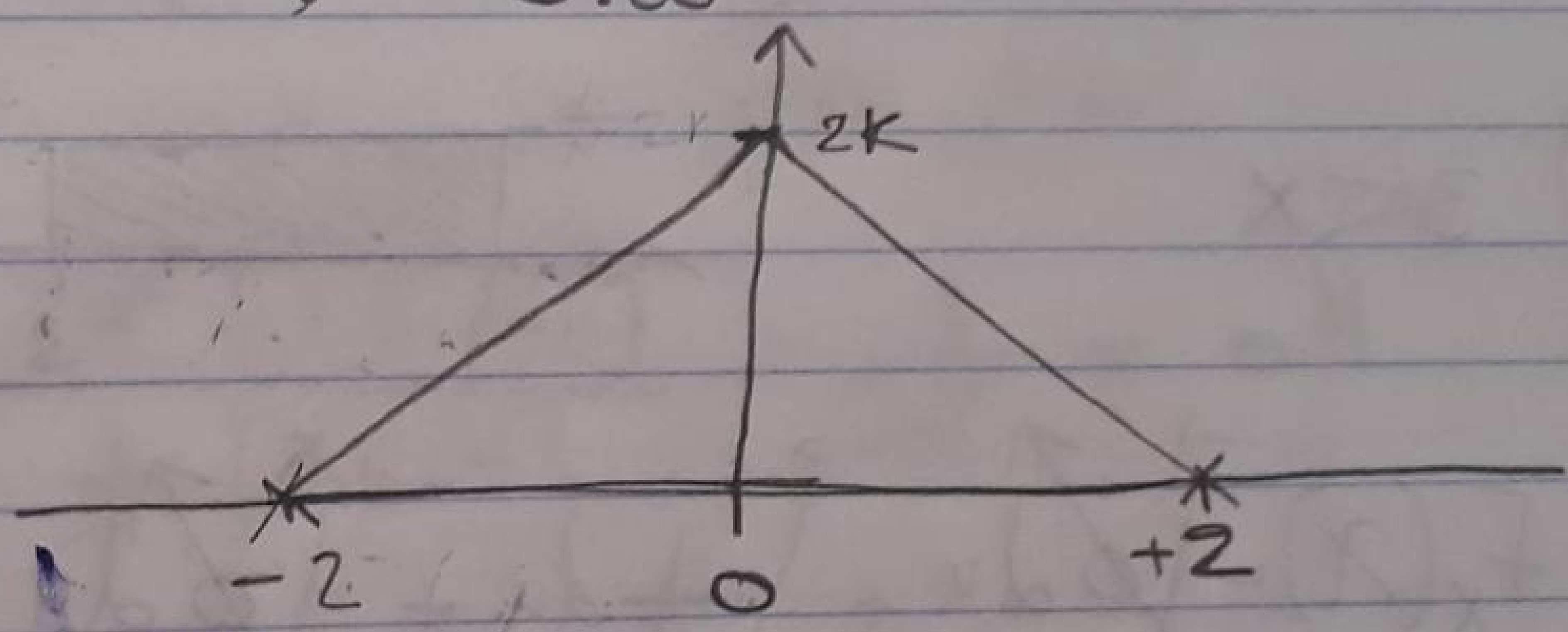
$$f_x(x) = \begin{cases} k(2-|x|) & , -2 \leq x \leq 2 \\ 0 & ; \text{O.W} \end{cases}$$

ⓐ Determine the value of K ?

we know that :-

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$f_x(x) = \begin{cases} k(2+x) & , -2 \leq x < 0 \\ k(2-x) & ; 0 \leq x < 2 \\ 0 & , \text{O.W} \end{cases}$$



عند تعويض -2 في المعادلة

$f_x(x) = 0$

عند تعويض +2 في المعادلة

$f_x(x) = 0$

عند تعويض 0 في المعادلة

$f_x(x) = 2k$ →

∴ الشكل الناتج هو مثلث متساوي الساقين

$$\therefore \int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$\therefore \int_{-2}^{0} k(2+x)\,dx + \int_{0}^{2} k(2-x)\,dx = 1$$

$$k\left(2x + \frac{x^2}{2}\right)\Big|_{-2}^{0} + k\left(2x - \frac{x^2}{2}\right)\Big|_{0}^{2} = 1$$

تكملة →

$$k\left[(0)-(-4+2)\right]+k\left[(4-2)-0\right]=1$$

$$2k+2k=1 \Rightarrow 4k=1 \Rightarrow \therefore \boxed{k=\frac{1}{4}}$$

حل آخر :- مساحة المثلث $= 1$

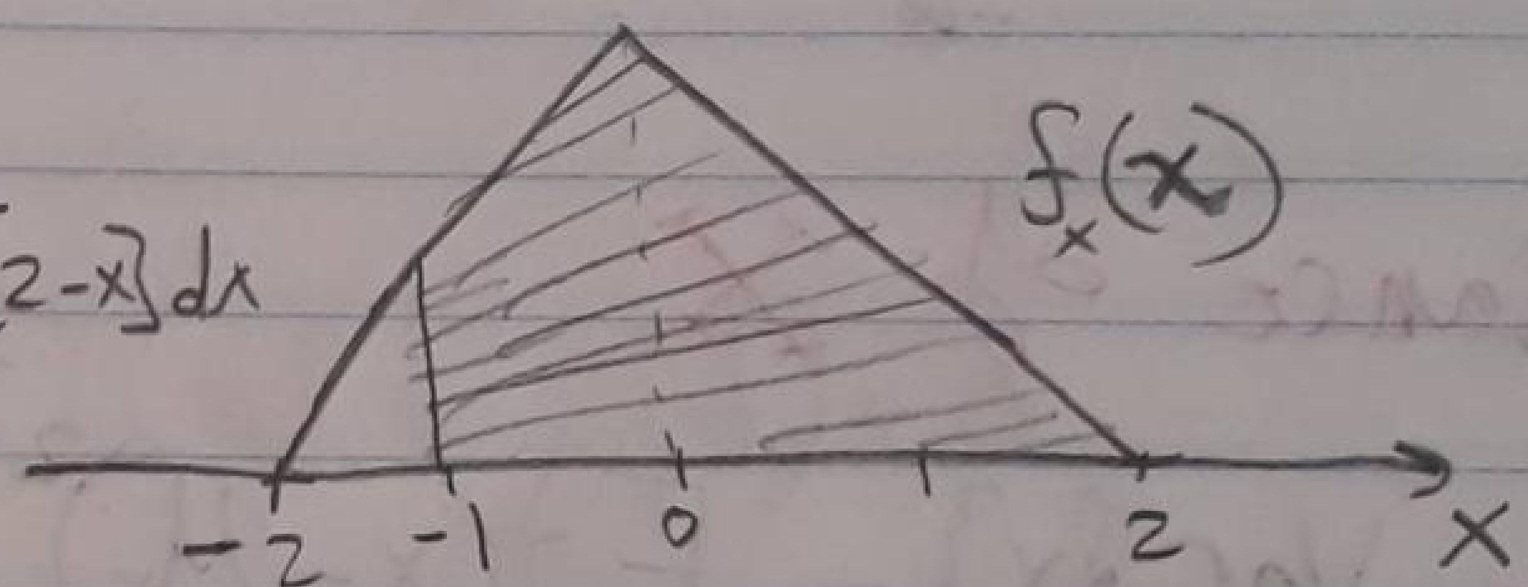
$\frac{1}{2} \times$ القاعدة $\times$ الارتفاع $= 1$

$$\therefore \frac{1}{2}\times 4\times 2k=1 \Rightarrow \boxed{k=\frac{1}{4}}$$

لا تُفضّل طريقة المساحة تحت المنحنى ، ممكن يبان الشكل بيكون
مش ــ دائرة ، مربع ، مستطيل ، ... ، ولكنه في الحقيقة غير ذلك ،
بالتالي بنغلط في الحسابات .

(b) $P(X \geq -1) = ?$

$$P(X\geq -1)=\int_{-1}^{0}\frac{1}{4}\left[2+x\right]dx+\int_{0}^{2}\frac{1}{4}\left[2-x\right]dx$$



OR:- $P(X\geq -1) = 1 - P(X<-1)$

$$=1-\int_{-2}^{-1}\frac{1}{4}\left(2+x\right]dx$$

$$=1-\frac{1}{4}\left[2x+\frac{x^2}{2}\right]\Bigg|_{-2}^{-1}$$

$$=1-\frac{1}{4}\left(\left[-2+\frac{1}{2}\right]-\left[-4+\frac{4}{2}\right]\right)$$

$$=1-\frac{1}{4}\left[\frac{-3}{2}+2\right] = 1-\frac{1}{4}\left(\frac{1}{2}\right)=1-\frac{1}{8}=\frac{7}{8}$$

* مشتقة الـ cdf هو الـ pdf .

Scanned by TapScanner

* Mean or Expected Value :-

إذا كان R.V دسكريت

$$\underbrace{E\{x\}}_{\text{Expected Value of } X \text{ OR Mean of } X} = \mu_x \overset{D}{=} \sum_{x=-\infty}^{\infty} x \underbrace{P(X=x)}_{\text{cdf}}$$

إذا كان R.V كونتينيوس

$$\overset{C}{=} \int_{-\infty}^{\infty} x \underbrace{f_X(x)}_{\text{pdf}} \, dx$$

$$\therefore E\{g(x)\} \overset{D}{=} \sum_{x=-\infty}^{\infty} g(x) \, P(X=x)$$

$$\overset{C}{=} \int_{-\infty}^{\infty} g(x) \, f_X(x) \, dx$$

* Variance of X

$$\sigma_x^2 = \mathrm{Var}\{x\} = E\{\underbrace{(x-\mu_x)^2}_{g(x)}\}$$

if discrete $\rightarrow \overset{D}{=} \sum_{x=-\infty}^{\infty} (x-\mu_x)^2 \, P(X=x)$

if continuous $\rightarrow \overset{C}{=} \int_{-\infty}^{\infty} (x-\mu_x)^2 \, f_X(x) \, dx$

* Standard deviation of X.

$$\sigma_x = \sqrt{\sigma_x^2}$$

$\rightarrow$ يعني لازم أطلع الvariance حتى أقدر أطلع الstandard deviation.

Ex Let X be a R.V with the following

$$P(X=x) = \begin{cases} \frac{1}{4} & x=-2 \\ \frac{1}{4} & x=-1 \\ \frac{1}{4} & x=1 \\ \frac{1}{4} & x=2 \\ 0 & \text{O.W} \end{cases}$$

Determine the ① μ_x ② σ_x^2 ③ σ_x

As the R.V is discrete, then:-

$$\mu_x = \sum_{x=-\infty}^{\infty} x \, P(X=x)$$

$$= \left[-2\left(\frac{1}{4}\right)\right] + \left(-1\left(\frac{1}{4}\right)\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 0$$

$$= \cancel{\frac{-1}{2}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{2}}$$

$$= 0$$

② $\sigma_x^2 = ?$

$$\sigma_x^2 = E\{(x-\mu_x)^2\}$$

$$= E\{(x-0)^2\}$$

$$= E\{x^2\} = \sum_{x=-\infty}^{\infty} x^2 P(X=x)$$

$$= (-2)^2\left(\frac{1}{4}\right) + (-1)^2\left(\frac{1}{4}\right) + (1)^2\left(\frac{1}{4}\right) + (2)^2\left(\frac{1}{4}\right) + 0$$

$$= 1 + \frac{1}{4} + \frac{1}{4} + 1 = 2.5$$

③ $\sigma_x = ?$

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{2.5} = 1.58114$$

ex $P(X = x) = \begin{cases} 1/4 & , x = -4 \\ 1/4 & , x = -3 \\ 1/4 & , x = -1 \\ 1/4 & , x = 0 \\ 0 & , \text{o.w} \end{cases}$

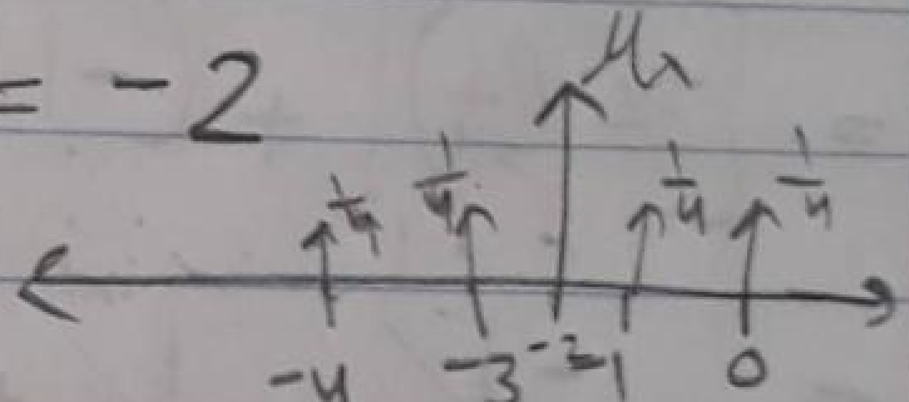
Find:

(a) μ_x (b) σ_x^2 (c) σ_x

$$\mu_x \overset{D}{=} \sum_{x=-\infty}^{\infty} x\, P(X = x)$$

$$= -4\left(\tfrac{1}{4}\right) + -3\left(\tfrac{1}{4}\right) + -1\left(\tfrac{1}{4}\right) + 0\left(\tfrac{1}{4}\right)$$

$$= \tfrac{1}{4}\left[-4 - 3 - 1 + 0\right] = \tfrac{1}{4} \cdot -8 = -2$$



(b) $\sigma_x^2 = E\{(X - \mu_x)^2\} = E\{\underbrace{(X+2)^2}_{g(x)}\}$

$$= \sum_{x=-\infty}^{\infty} (x+2)^2\, P(X = x)$$

$$= (\overset{-2}{-4+2})^2\left(\tfrac{1}{4}\right) + (\overset{-1}{-3+2})^2\left(\tfrac{1}{4}\right) + (-1+2)^2\left(\tfrac{1}{4}\right) + (0+2)^2\left(\tfrac{1}{4}\right) + 0$$

$$= \tfrac{4}{4} + \tfrac{1}{4} + \tfrac{1}{4} + 1 + 0$$

$$= 2.5$$

(c) $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{2.5} = 1.58114$

ex3 $P(X=x) = \begin{cases} 1/4 & , x=-4 \\ 1/4 & , x=-2 \\ 1/4 & , x=2 \\ 1/4 & , x=4 \\ 0 & , \text{o.w} \end{cases}$

Find:-

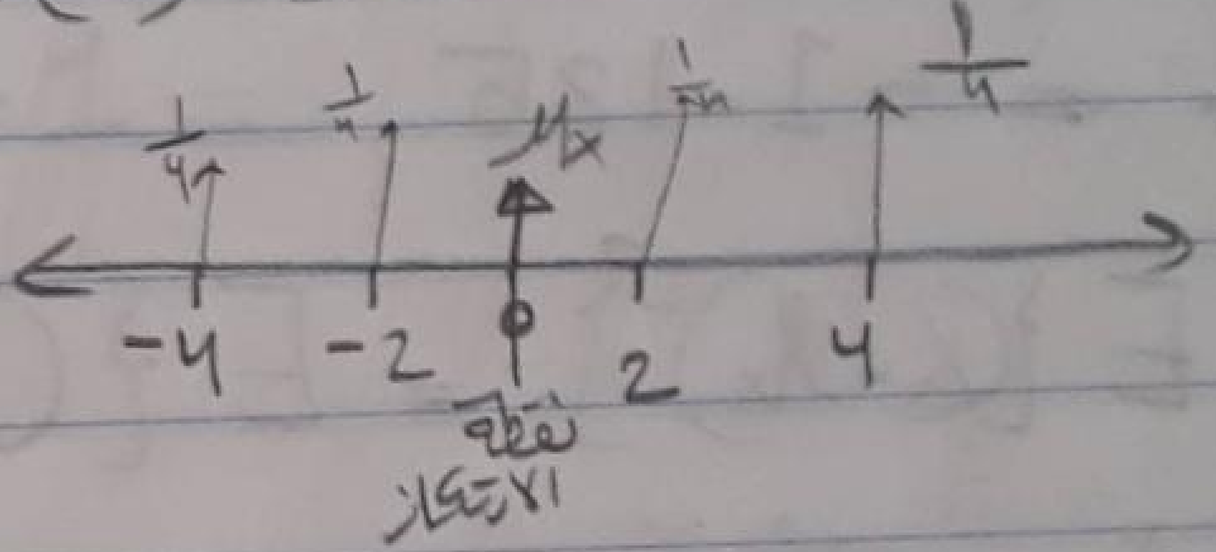
(a) μ_x , (b) σ_x^2 , (c) σ_x

* وحدة الـ μ_x زي وحدة x ، إذا كانت متر بتكون متر ، أما الـ $P(X=x)$ ما إلها وحدة ، و الـ σ_x^2 وحدتها زي x ولكن مربعة عشان μ_x مربعة.

(a) $\mu_x \stackrel{D}{=} E\{X\} = \sum_{x=-\infty}^{\infty} x\,P(X=x)$

$= -4\left(\frac{1}{4}\right) + -2\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right)$

$= \frac{1}{4}\left[\cancel{-4}\cancel{-2}+\cancel{2}+\cancel{4}\right] = \frac{1}{4}(0) = \text{zero}$



(b) $\sigma_x^2 = E\{(X-\mu_x)^2\}$

$= E\{(X-0)^2\} = E\{X^2\} = \sum_{n=-\infty}^{\infty} x^2\,P(X=x)$

$= \overset{16}{(-4)^2}\left(\frac{1}{4}\right) + \overset{4}{(-2)^2}\left(\frac{1}{4}\right) + (2)^2\left(\frac{1}{4}\right) + (4)^2\frac{1}{4} + 0$

$= 4 + 1 + 1 + 4 + 0$

$= 10$

يشير الـ Variance إلى مقدار التشتت عن الـ mean ، يعني قديش كل قيمة بعيدة عن الـ mean value.

(c) $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{10} = 3.16228$

فعند مقارنة القيم في المثالين الماضيين و المثال هذا نلاحظ أنّه هنا القيم ابتعدت أكثر (تشتت) عن الـ mean.

Example :-

$$P(X=x) = \begin{cases} 1/2 & , x=-2 \\ 1/4 & , x=-1 \\ 1/8 & , x=0 \\ 1/8 & , x=1 \\ 0 & , \text{o.w} \end{cases}$$

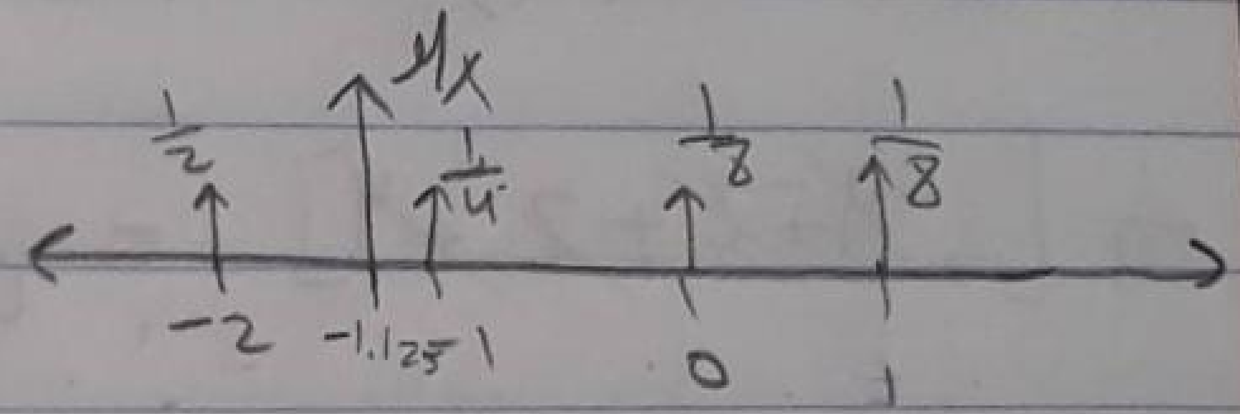
Find:- (a) μ_x
(b) σ_x^2
(c) σ_x

(a) $\mu_x \triangleq E\{X\} = \sum_{n=-\infty}^{\infty} x\, P(X=x)$

$= \overbrace{(-2)(\tfrac{1}{2})}^{-1} + (-1)(\tfrac{1}{4}) + 0 + \tfrac{1}{8} + 0$

$= -1 - \frac{1}{4} + \frac{1}{8}$

$= -\frac{9}{8} = -1.125$



(b) $\sigma_x^2 = E\{(X-\mu_x)^2\} = E\{(X+\frac{9}{8})^2\}$

أول ما أشيل E أحط P

$= \sum_{n=-\infty}^{\infty} (x+\frac{9}{8})^2 P(X=x)$

$= (-2+\frac{9}{8})^2(\frac{1}{2}) + (-1+\frac{9}{8})^2(\frac{1}{4}) + (\frac{9}{8})^2(\frac{1}{8}) + (1+\frac{9}{8})^2(\frac{1}{8})$

$= 0.383 + 3.90625\times10^{-3} + 0.1582 + 0.5644$

$= 1.10938$

(c) $\sigma_x = \sqrt{\sigma_x^2} = 1.05327$

Scanned by TapScanner

Ex) Let X be a R.V with the following pdf

$$f_x(x) = \begin{cases} \overset{3/4}{k}(1-x^2) & , -1 \le x \le 1 \\ 0 & , \text{o.w} \end{cases}$$

a) Determine the value of k?

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$\int_{-1}^{1} k(1-x^2) = 1 \Rightarrow k\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = 1$$

$$k\left[1-\frac{1}{3}\right] - k\left[-1+\frac{1}{3}\right] = 1$$

$$k - \frac{1}{3}k + k - \frac{1}{3}k = 1$$

$$3.\frac{2}{3}k - \frac{2}{3}k = 1 \Rightarrow \frac{6}{3}k - \frac{2}{3}k = 1 \Rightarrow \frac{4}{3}k = 1$$

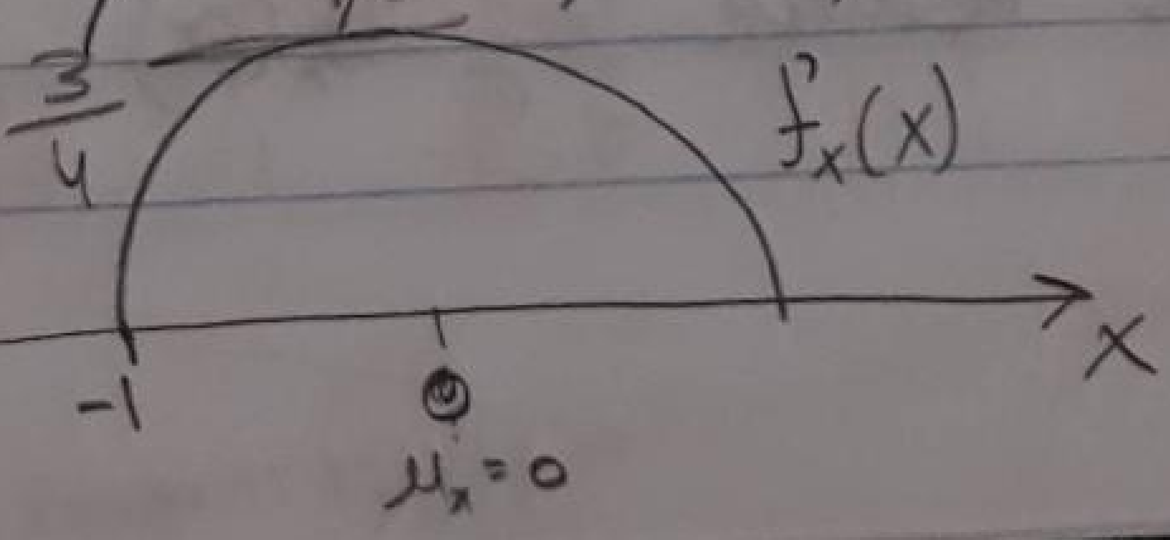
$$\boxed{k = \frac{3}{4}}$$

b) Determine the expected value of X.

$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\,dx = \int_{-1}^{1} x \cdot \left[\frac{3}{4}(1-x^2)\right]dx$$

$$= \int_{-1}^{1} \frac{3}{4}x - \frac{3}{4}x^3\,dx = \frac{3}{4(2)}x^2 - \frac{3}{4(4)}x^4 \Big|_{-1}^{1}$$

$$= \left(\frac{3}{8} - \frac{3}{16}\right) - \left[\frac{3}{8} - \frac{3}{16}\right] = \cancel{\frac{3}{8}} - \cancel{\frac{3}{16}} - \cancel{\frac{3}{8}} + \cancel{\frac{3}{16}} = 0$$



عرفنا إنو رسمناها دائرة مش مثلث أو خط من المعادلة مخروق $(1-x^2)$

(c) $\sigma_x^2 = ??$

$$\sigma_x^2 = E\{(x-\mu_x)^2\} = E\{(x-0)^2\} = E\{x^2\}$$

$$= \int_{-\infty}^{\infty} x^2 f_x(x)\, dx$$

$$= \int_{-1}^{1} x^2 \left[\frac{3}{4}(1-x^2)\right] . dx$$

$$= \int_{-1}^{1} \left(\frac{3}{4}x^2 - \frac{3}{4}x^4\right) dx$$

$$= \frac{\cancel{3}}{4(\cancel{3})}x^3 - \frac{3}{4(5)}x^5 \Big|_{-1}^{1} = \left[\frac{1}{4} - \frac{3}{20}\right] - \left[\frac{-1}{4} + \frac{3}{20}\right]$$

$$= \frac{1}{4} - \frac{3}{20} + \frac{1}{4} - \frac{3}{20}$$

$$= \frac{2}{4} - \frac{\cancel{6}^{3}}{\cancel{20}_{10}} = \frac{1}{2} - 0.3 = 0.2$$

(d) $\sigma_x = ??$

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{0.2} = 0.44721$$

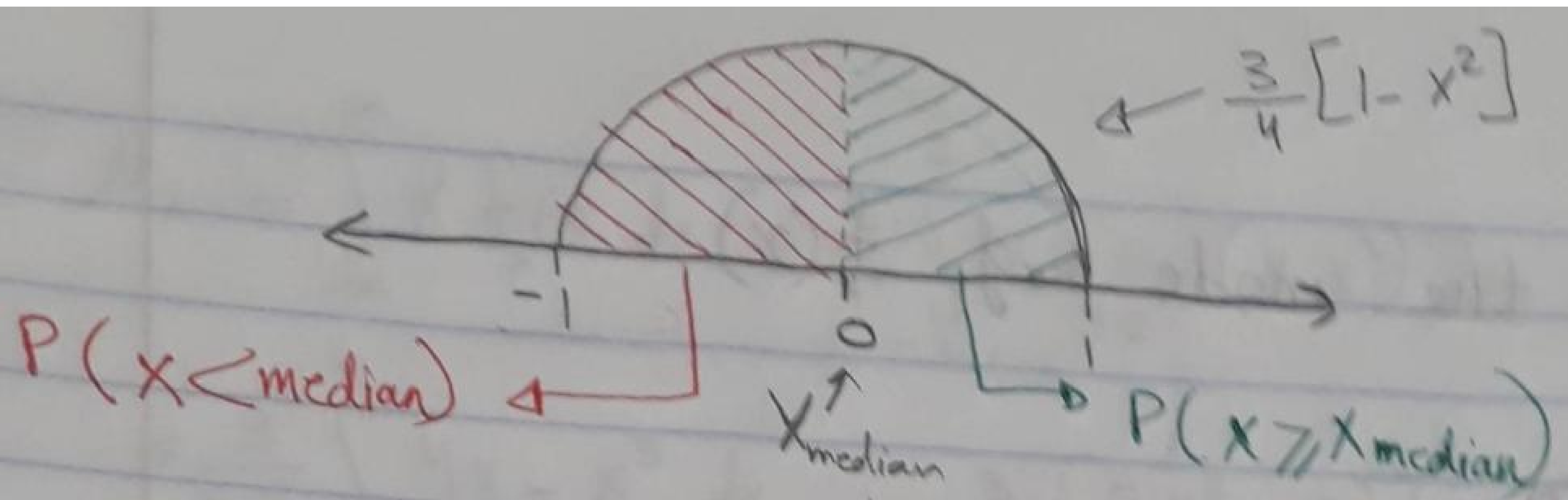
(e) Determine the median of $f_x(x)$? ← فقط خاص بال Pdf

$(X_{median} = X_0)$ is the median of $f_x(x)$.

$$P(X < X_{median}) = P(X \geq X_{median})$$

X_{median} is the median of $f_x(x)$

تابع →



هي النقطة الي تقسم ال curve لـ منطقتين متساويين .

$$\therefore P(X < X_{median}) + P(X \geq X_{median}) = 1 \leftarrow$$

Probability ال لكل ال Sample space = 1

Sample space ال تشكل

$$\therefore P(X < X_{median}) + P(X < X_{median}) = 1$$

$$\therefore \boxed{P(X < X_{median}) = \frac{1}{2}}$$

How to calculate x_0 (X_{median})?

$$\rightarrow \int_{-\infty}^{x_0} f_x(x) \cdot dx = \frac{1}{2}$$

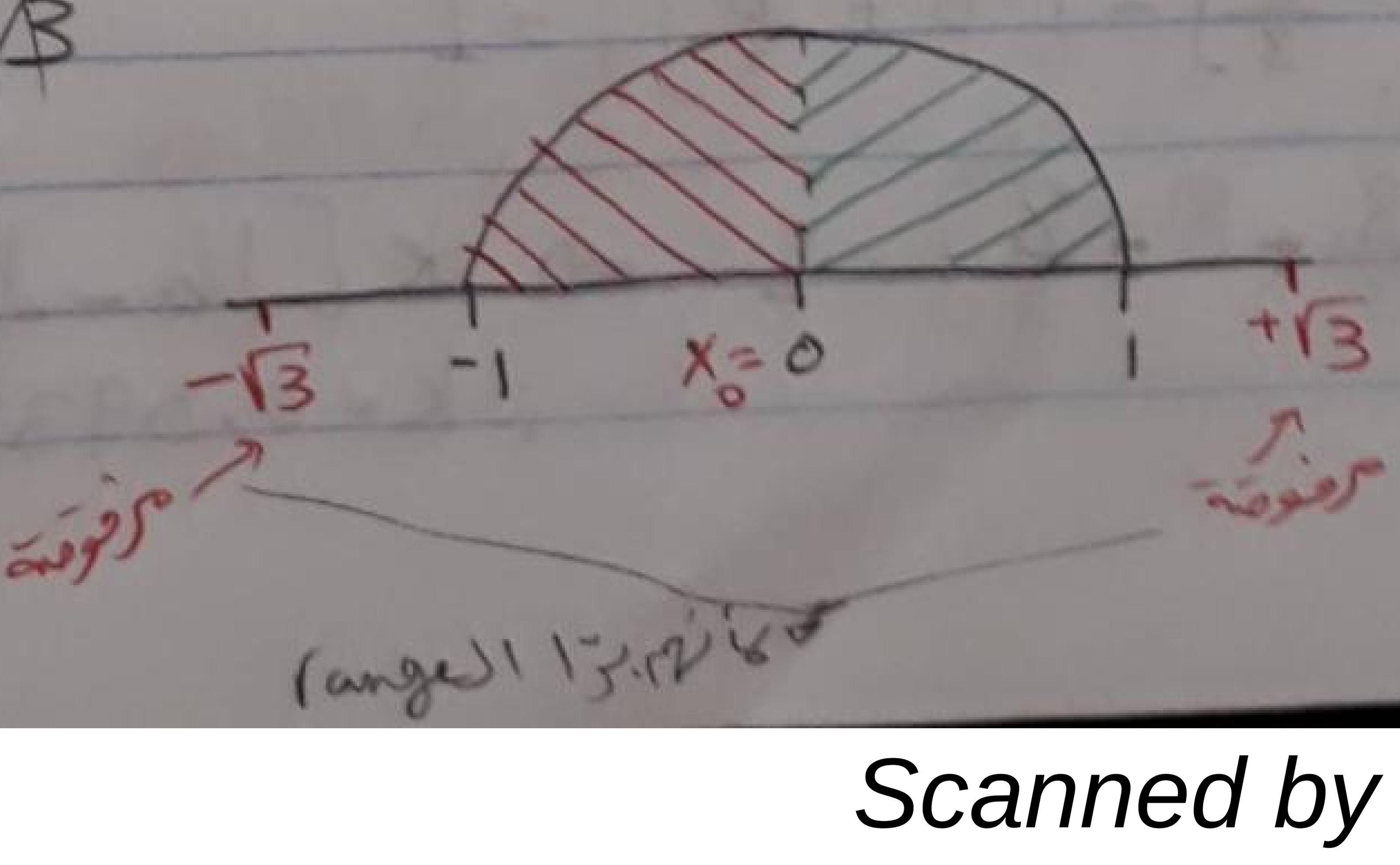
$$\therefore \int_{-1}^{x_0} \frac{3}{4}[1-x^2] \cdot dx = \frac{1}{2}$$

$$\therefore \frac{3}{4}\left[x - \frac{x^3}{3}\right]_{-1}^{x_0} = \frac{3}{4}\left[\left(x_0 - \frac{x_0^3}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = \frac{1}{2}$$

$$\frac{3}{4}x_0 - \frac{1}{4}x_0^3 + \cancel{\frac{3}{4}} - \cancel{\frac{1}{4}} = \cancel{\frac{1}{2}}$$

$$\frac{3}{4}x_0 - \frac{1}{4}x_0^3 = 0 \Rightarrow x_0\left[\frac{3}{4} - \frac{1}{4}x_0^2\right] = 0$$

$$x_0 = 0, \cancel{\sqrt{3}}, \cancel{-\sqrt{3}}$$



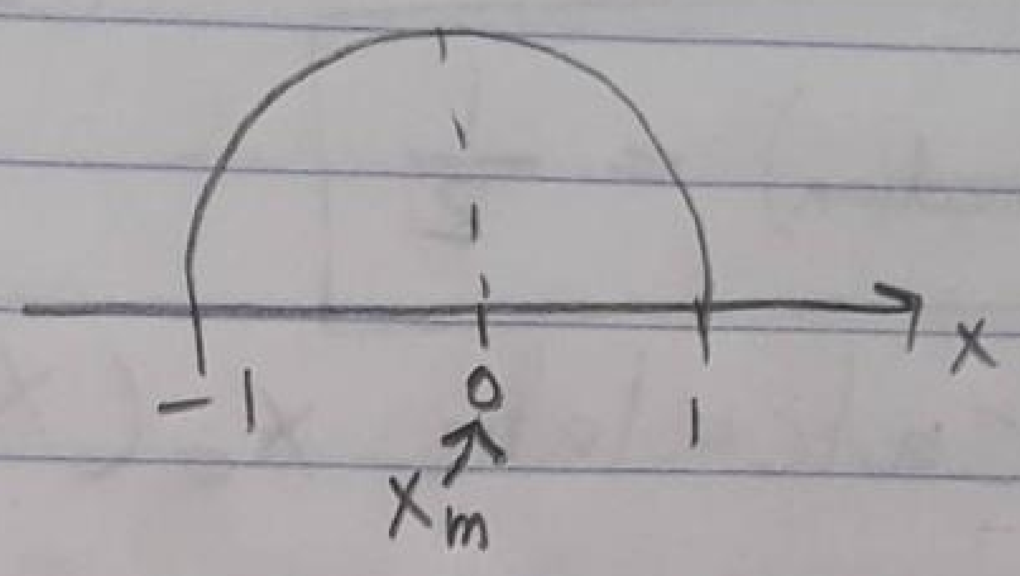
(f) Determine the mode of $f_x(x)$

$$x_{mode} = x_m$$

$f_x(x_m) = \max |f_x(x)|$ → قيمة x الي بتخلي $f_x(x)$ تكون Maximum هي ال mode of X .

$$\frac{df_x(x)}{dx} = 0 \Rightarrow \frac{d\left[\frac{3}{4}(1-x^2)\right]}{dx} = \frac{3}{4}(-2x) = 0$$

$$\therefore \boxed{x_m = 0}$$



في هاد السؤال فقط صدفة طلع إنو ال

Mean يساوي Median يساوي mode of x

. فقط صدفة

Exercise #1

$$f_x(x) = \begin{cases} k(4-x^2), & -2 \le x \le 2 \\ 0, & \text{o.w} \end{cases}$$

Find ① μ_x ② σ_x^2 ③ σ_x

First we find k; $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$

$$\therefore \int_{-2}^{2} k(4-x^2)\,dx = 1 \Rightarrow k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left(\left[8 - \frac{8}{3}\right] - \left[-8 + \frac{8}{3}\right]\right) = 1$$

$$k\left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right] = 1 \Rightarrow k\left[16 - \frac{16}{3}\right] = 1$$

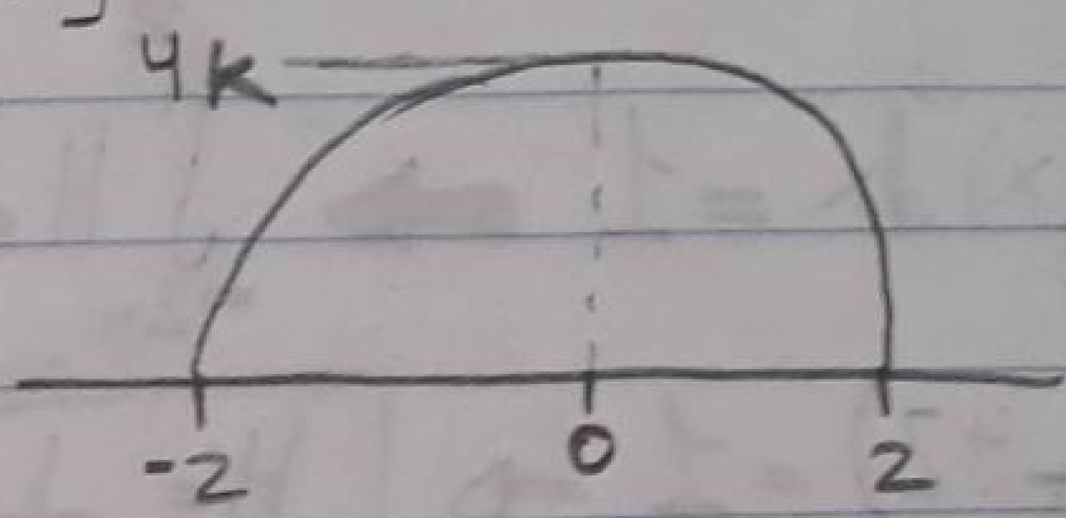
$$\therefore k = 0.09375 = \frac{3}{32}$$

Scanned by TapScanner

$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\, dx = \int_{-2}^{2} x * \frac{3}{32}(4-x^2)\, dx$$

$$= \frac{3}{32}\int_{-2}^{2} 4x - x^3\, dx = \frac{3}{32}\left[2x^2 - \frac{x^4}{4}\right]_{-2}^{2}$$

$$= \frac{3}{32}\left[\cancel{\left(8 - \frac{16}{4}\right)} - \cancel{\left(8 - \frac{16}{4}\right)}\right] = 0$$



(2) $$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2\, f_x(x)\, dx$$

$$= \int_{-2}^{2} (x-0)^2 * \frac{3}{32}(4-x^2)\, dx = \frac{3}{32}\int_{-2}^{2} x^2(4-x^2)\, dx$$

$$= \frac{3}{32}\int_{-2}^{2} 4x^2 - x^4\, dx = \frac{3}{32}\left[\frac{4}{3}x^3 - \frac{x^5}{5}\right]_{-2}^{2}$$

$$= \frac{3}{32}\left(\left[\frac{4}{3}(8) - \frac{32}{5}\right] - \left[-\frac{4}{3}(8) + \frac{32}{5}\right]\right)$$

$$= \frac{3}{32}\left[\frac{32}{3} - \frac{32}{5} + \frac{32}{3} - \frac{32}{5}\right]$$

$$= \frac{3}{32}\left[\frac{64}{3} - \frac{64}{5}\right]$$

$$= 0.8$$

(3) $$\sigma_x = \sqrt{\sigma_x^2} = 0.89443$$

Exercise #2

$$f_x(x) = \begin{cases} H & , -2 \le x \le 2 \\ 0 & , \text{o.w} \end{cases}$$

Find :
1. μ_x
2. σ_x^2
3. σ_x

1. $\mu_x = ??$ First, we find H :-

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1 \Rightarrow \int_{-2}^{2} H\, dx = 1$$

$$H(2 - -2) = 1 \Rightarrow \boxed{H = \frac{1}{4}}$$

$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\, dx = \int_{-2}^{2} x \left(\frac{1}{4}\right) dx = \left(\frac{1}{4}\right) \frac{x^2}{2}\Big|_{-2}^{2}$$

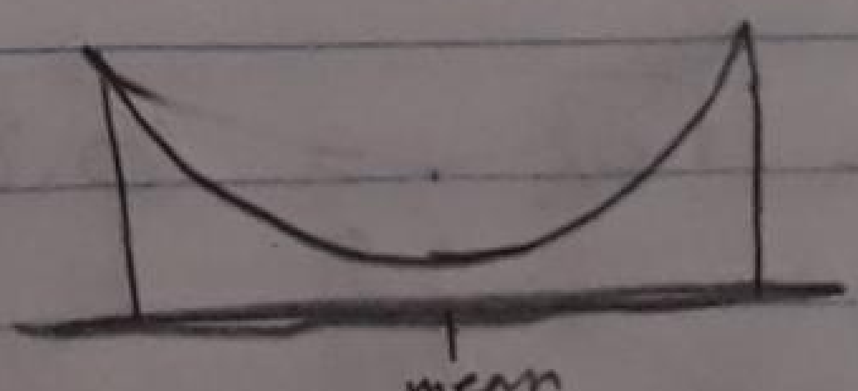
$$= \frac{1}{4}\left[\frac{4}{2} - \frac{4}{2}\right] = 0$$

2. $$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x)\, dx$$

$$= \int_{-2}^{2} (x - 0)^2 f_x(x)\, dx = \int_{-2}^{2} x^2 f_x(x)\, dx$$

$$= \int_{-2}^{2} x^2 \cdot \frac{1}{4}\, dx = \frac{1}{4} \frac{x^3}{3}\Big|_{-2}^{2} = \frac{1}{4}\left[\frac{8}{3} + \frac{8}{3}\right] = \frac{1}{\cancel{4}} \cdot \frac{\cancel{16}^{\,4}}{3} = \frac{4}{3}$$

3. $$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$



التشتت بهاي بيكون أكبر من اللي فوقها ، عشان
كل ما نبعد القيم عن الـ mean ، رح يزيد التشتت
والتشتت m بـ ex1 أقل من ex2 عشان القيم متجمعين
الـ mean أكثر من حواليه ، زي جبل الرمل لما القيم
تتجمع بالبعض .

Exercise#3

$$f_x(x) = \begin{cases} kx & , \; 0 \leq x \leq 1 \\ 0 & , \; \text{o.w} \end{cases}$$

Find:- ① μ_x ② σ_x^2 ③ σ_x ④ Median of $f_x(x)$.

⑤ Mode of $f_x(x)$.

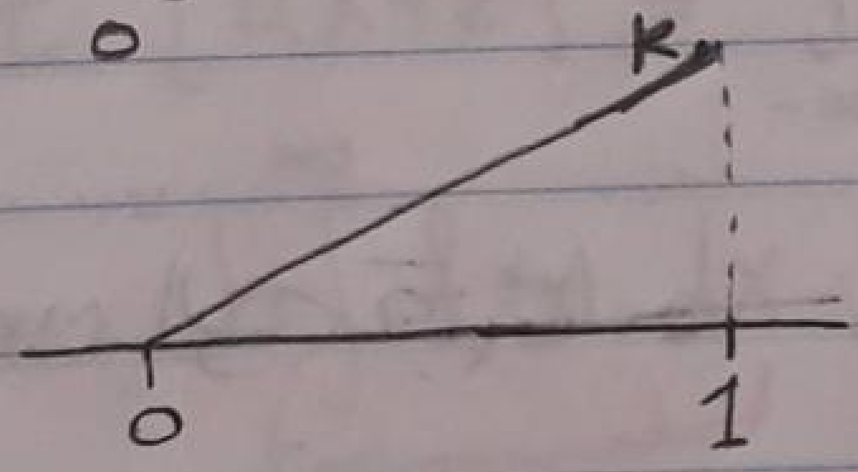
First, find k :- $\int_{-\infty}^{\infty} f_x(x)\, dx = 1$

$\therefore \int_0^1 kx\, dx = 1 \Rightarrow k \frac{x^2}{2} \Big|_0^1 = 1$

$\frac{k}{2} = 1 \Rightarrow \boxed{k=2}$

① $\Rightarrow \mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\, dx = \int_0^1 x \cdot 2x\, dx = \int_0^1 2x^2 dx = \frac{2}{3} x^3 \Big|_0^1$

$= \frac{2}{3}$



② $\sigma_x^2 = E\{(x-\mu_x)^2\} = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_x(x)\, dx$

$= \int_0^1 (x-\frac{2}{3})^2 (2x)\, dx = \int_0^1 (x^2 - \frac{4}{3}x + \frac{4}{9}) 2x\, dx$

$= \int_0^1 2x^3 - \frac{8}{3}x^2 + \frac{8}{9}x\, dx$

$= \frac{2x^4}{4} - \frac{8x^3}{9} + \frac{8x^2}{18} \Big|_0^1$

$= \frac{1}{2} \underbrace{- \frac{8}{9} + \frac{4}{9}}_{} - 0 = \frac{1}{2} - \frac{4}{9} = 0.055556$

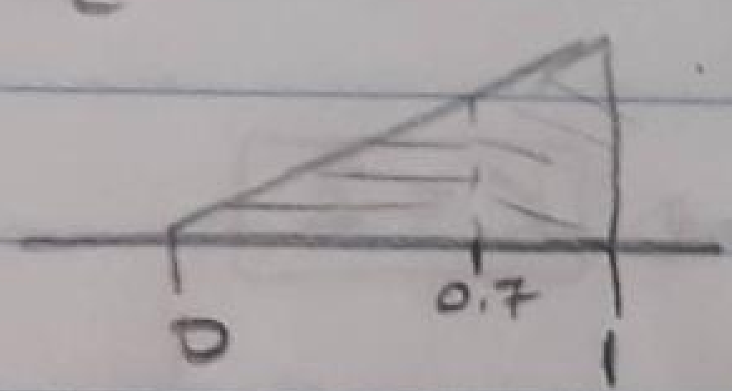
③ $\sigma_x = \sqrt{\sigma_x^2} = 0.23570226$

④ Median of $f_x(x)$

$$\int_{-\infty}^{x_0} f_x(x)\,dx = \frac{1}{2}$$

$$\int_{0}^{x_0} 2x\,dx = \frac{1}{2}$$

$$\therefore x^2 \Big|_0^{x_0} = \frac{1}{2} \Rightarrow x_0^2 - 0 = \frac{1}{2} \Rightarrow x_0^2 = \frac{1}{2}$$



$$\therefore x_0 = \pm \frac{1}{\sqrt{2}}$$

$$\therefore \boxed{x_0 = 0.707} \in [0,1]$$

⑤ Mode of $f_x(x)$

$\boxed{\text{Mode of } x = 1}$ من الرسمة :

هي النقطة اللي خلت $f_x(x)$ أعلى إشي .

دائماً إزا ما قدرت أساوي المشتقة بال zero بفضل إني أفحص
الأطراف ، زي هاد المثال ، مستحيل إنو $2=0$ ، عشان هيك بنفحص
الأطراف وبنشوف عند ال 1 و ال zero .

$\therefore f_x(0) = 0$

$f_x(1) = 2 \rightarrow$ Maximum $\rightarrow \therefore \boxed{\text{Mode of } f_x(x) = 1}$

Theorem:- Let X be a random variable with mean μ_X and variance σ_X^2.

Define $Y = aX + b$; (a) and (b) are real constants, then:-

① $\mu_y = a\mu_x + b$

② $\sigma_y^2 = a^2 \sigma_x^2$

Ex: let X be a R.V with mean of 2 and variance of 4. $Y = 3X - 5$ is a new R.V. Determine the mean and variance of Y?

Note:- $Y = ax + b$ $\longrightarrow$ بس شان نوضح من وين إجوا المعادلتين فوق

$$\mu_y = E\{y\} = E\{ax+b\} \stackrel{c}{=} \int_{-\infty}^{\infty} (ax+b)\, f_x(x)\, dx$$

$$= a\left[\int_{-\infty}^{\infty} x\, f_x(x)\right] dx + b\underbrace{\int_{-\infty}^{\infty} f_x(x)\, dx}_{1}$$

$$= a\mu_x + b * 1$$

$$\boxed{\mu_y = a\mu_x + b}$$

$$\sigma_y^2 = E\{(Y - \mu_y)^2\} = E\{(ax + \cancel{b} - a\mu_x - \cancel{b})^2\}$$

$$= E\{(ax - a\mu_x)^2\} = E\{a^2 (x - \mu_x)^2\}$$

الـ E زي التكامل بتقدر فيها أطلع الـ constant "a" مرة زي برا .

$$= a^2 \underbrace{E\{(x - \mu_x)^2\}}_{\sigma_x^2}$$

$$\boxed{\sigma_y^2 = a^2 \sigma_x^2}$$

حل المثال $\longrightarrow$

Scanned by TapScanner

$$\mu_y = E\{y\} = E\{3x-5\} = 3E\{x\}-5$$

$$= (3*2)-5 = 1$$

صعب نحلها

$$\sigma_y^2 = E\{(y-\mu_y)^2\} = E\{(3x-5-1)^2\} = E\{(3x-6)^2\} = \ldots$$

فنحل بالقانون دغري أسهل

$$= a^2\sigma_x^2 = (3)^2 * 4 = 9\times 4 = 36$$

Note: $\sigma_x^2 = E\{(x-\mu_x)^2\}$

$$= E\{x^2 - 2x\mu_x + (\mu_x)^2\}$$

ال E لأي رقم هو الرقم نفسه

$$= E\{(x^2)\} - 2\mu_x \underbrace{E\{x\}}_{\mu_x \text{ نفسه}} + (\mu_x)^2$$

$$\therefore \sigma_x^2 = E\{x^2\} - 2\mu_x^2 + \mu_x^2$$

$$= E\{x^2\} - \mu_x^2$$

$$\therefore \boxed{E\{x^2\} = \sigma_x^2 + \mu_x^2}$$

تبعاً لهذا القانون، صرنا نقدر نحل هاي بشكل أسهل.

$$E\{9x^2 - 36x + 36\} = 9E\{x^2\} - 36E\{x\} + 36$$

$$= 9[\sigma_x^2 + \mu_x^2] - 36\mu_x + 36$$

$$= 9[\overset{8}{4+(2)^2}] - (36*2) + 36$$

$$= 72 - 36$$

نفس الجواب

$$\therefore \boxed{\sigma_y^2 = 36}$$

Scanned by TapScanner

Ex Let R be a random variable with $\mu_R = 0.5$, $\sigma_R = 2$, $y = 2R + 3$ ⓐ Determin the mean and Standard deviation of y?

$$\mu_y = a\mu_R + b$$
$$= 2(0.5) + 3$$
$$= 4$$

← طريقة ① عالقانون

طريقة ②

$$\mu_y = E\{y\} = E\{2R+3\} = 2\underbrace{E\{R\}} + 3 = 2\mu_R + 3$$
$$= 2(0.5) + 3$$
$$= 4$$

$$\sigma_y^2 = a^2 \sigma_R^2 \rightarrow$$
$$= 4(2)^2$$
$$= 16$$

لاحظ أنّ المُعطى هو σ_R وليس σ_R^2

$$\sigma_y = \sqrt{\sigma_y^2} = \sqrt{16} = 4$$

← دائماً هو الجذر الموجب للـ variance

ⓑ $G = 2R^2 - 3R + 1$, Determine the mean of G.

ممنوع أطلع التربيع برا وأحول لإنها تساوي $\cancel{(E\{R\})^2}$

$$\mu_G = E\{G\} = E\{2R^2 - 3R + 1\} = 2E\{R^2\} - 3E\{R\} + 1$$

$$= 2E\{R^2\} - 3\mu_R + 1$$

باستخدام الـ Note السابقة:-

$$= 2[\sigma_R^2 + \mu_R^2] - 3\mu_R + 1$$

$$= 2\left[4 + \frac{1}{4}\right] - \frac{3}{2} + 1 = 8 + \frac{1}{2} - \frac{3}{2} + 1 = 9 - 1$$
$$= 8$$

Note:- $E\{R^3\} \neq E\{R^2\}\,E\{R\}$

$$\int x^3 dx \neq \int x^2 dx \int x\, dx$$

← نفس مبدأ التكامل :

يعني أنّ E لا تتوزع على الضرب تماماً كما لا يتوزع التكامل على الضرب

Common Discrete Random Variables:-

1 The Binomial Distribution

A Random experiment consisting of (n) repeated trials, such that:-

a. The trials are independent.

b. Each trial results in only two possible outcomes, a success and a failure.

c. The probability of a success (P) on each trial remains constant.

Is called a binomial experiment.

ex) Consider the experiment of flipping the coin for three times. Assume $P(H) = 1/4$, and $P(T) = 3/4$.

a) Determine the probability of getting head for 2 times.

$n = 3$, $\overset{\text{Random variable}}{X}$: Number of heads (success) in $n(3)$ trials.

$$P(X = x) = \binom{n}{x} (P(s))^{x} \left[\underbrace{1 - P(s)}_{\text{failure}}\right]^{n-x} \quad , x = 0, 1, \dots, n.$$

الاحتمالية تبعت الـ failure

$$P(X = x) = \binom{3}{x} \left(\frac{1}{4}\right)^{x} \left(\frac{3}{4}\right)^{3-x}$$

$$P(X = x) = \begin{cases} \binom{3}{x} \left(\frac{1}{4}\right)^{x} \left(\frac{3}{4}\right)^{3-x} & , x = 0, 1, \dots, 3 \\ 0 & , \text{o.w} \end{cases}$$

Scanned by TapScanner

$$P(x=2) = \binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^1 = 3 * \frac{1}{16} * \frac{3}{4} = \frac{9}{64}$$

(b) What is the probability of getting at least one head?

$$P(X=x) = \binom{3}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{3-x}, \quad x = 0, 1, 2, 3$$

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$$

$$\text{OR} = 1 - P(X<1) = 1 - P(X=0)$$

$$= 1 - \binom{3}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3$$

$$= 1 - \frac{27}{64}$$

$$= \frac{37}{64} \quad \#$$

يعني a معنى هاد الحكي :-

S (sample space)

TTT
TTH
THT
HTT

$$\left.\begin{array}{l} \text{HHT} \rightarrow (P(H))^2 (P(T)) = (1/4)^2 * \left(\frac{3}{4}\right) \\ \text{HTH} \rightarrow (P(H))^2 (P(T)) = (1/4)^2 * \left(\frac{3}{4}\right) \\ \text{THH} \rightarrow (P(H))^2\, P(T) = (1/4)^2 * (3/4) \end{array}\right\}$$

HHH

$$3 * \left(\frac{1}{4}\right)^2 * \frac{3}{4} = \frac{9}{64} \rightarrow$$ نفسه جواب فرع a

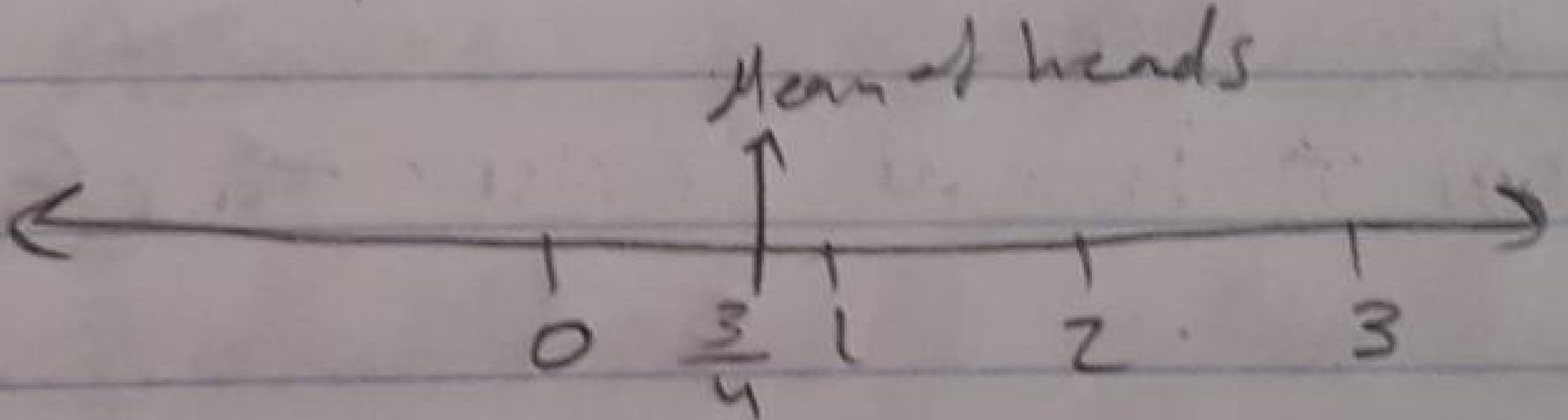
(c) What is the expected number of heads to be observed in the experiment?

$\boxed{\text{Theorem}}$: If (X) is a binomial r.v (random variable) with Parameteres (n) and (P), then:

$$M_x = E\{(X)\} = nP \quad \text{(نسبة ال success)}$$

$$\sigma_x^2 = Var(X) = n\,P(1-P)$$

$$M_x = E\{x\} = n\,P(s) = 3 * \frac{1}{4} = \frac{3}{4}$$ → معناها إنو عملنا التجربة بنعمل رمي ال coin ٣ رميات، للتجربة الواحدة، مثلاً رميناهم ١٠٠ مرة فمعناها إنو بمعدل ال 100 مرة، كان يطلع head بس نسبته $\frac{3}{4}$، يعني بنقدر نقول جراوح بين 1 head وال 2 الباقيين T أو 0 head، و 3 T. بس ال $\frac{3}{4}$ أقرب لل 1، فهيك ال Mean عدد مرات ظهور ال heads في



(D) find the variance of number of heads observed?

$$\sigma_x^2 = n\,P(s)\,[1-P(s)] = 3 * \frac{1}{4} * \frac{3}{4} = \frac{9}{16}$$

Ex Consider the experiment of tossing a dice for $\boxed{3\text{ times}}$, (a) what is the Probability of getting a number less than 2 for one time.

$S_{\text{one trial}} \rightarrow \{1, 2, 3, 4, 5, 6\}$

success $= \{1\} \rightarrow P(S) = \frac{1}{6}$

Failure $= \{2, 3, 4, 5, 6\}$

$= P(F) = \frac{5}{6}$

$\therefore X$: Number of trials while observing number less than 2.

تحقق شروط ال binomial ← اذاً $\therefore$

$$P(X=x) = \binom{n}{x} (P(S))^x \left[1-P(S)\right]^{n-x}$$

$$= \binom{3}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}, \quad x = 0, 1, 2, 3$$

$$P(x=1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$$

(b) What is the probability of getting a number divisible by 3 for $\boxed{2 \text{ times}}$

3,6

$S = \{1, 2, 3, 4, 5, 6\}$

Success $= \{3, 6\} \rightarrow P(S) = \frac{2}{6} = \frac{1}{3}$

Failure $= \{1, 2, 4, 5\} \rightarrow P(F) = \frac{4}{6} = \frac{2}{3}$

$n = 3$ ← [illegible]

X: number of trials while observing a number divisible by 3.

$$P(X=x) = \binom{n}{x} (P(S))^x \left[1-P(S)\right]^{n-x}$$

$$= \binom{3}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{3-x}, \quad x = 0, 1, 2, \underset{n}{3}$$

$$P(x=2) = \binom{3}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1$$

[illegible]

$$= \frac{\cancel{3!}^{\,3}}{\cancel{(3-2)}!\,2!} * \frac{1}{9} * \frac{2}{\cancel{3}} = \frac{2}{9}$$

The Geometric Distribution

هو experiment تتكرر ، لكنها تتكرر إلى مالانهاية ، ليس إلى عدد معين
من المرات ، ونفس ال binomial في لازم two outcomes ، و independent

Probability of failure و Probability of success .

والـ R.V (X) بيمثل ال Number of trials to the first success

$$\underbrace{\underset{1}{F}\ \underset{2}{F}\ \underset{3}{F}\ \underset{4}{F}\ \underset{5}{S}}\ \ldots$$

X: number of trials to the first success

$$P(X=5) = P(F)^4\, P(S)$$

شو احتمالية أحصل على success لأول مرة ، بعد ما أعمل 5 تجارب وطلعوا كلهم failure إلا الأخيرة
مش معناها شو ال P إنو يطلع success بالمرة الخامسة .
لا ، إنها يطلع (S) لأول مرة بعد ما جربت 5 مرات

$$\therefore P(X=x) = P(F)^{x-1}\, P(S)$$

كيف بنكتب ال Geometric ال cdf :-

$$P(X=x) = \begin{cases} \underbrace{(1-P)}^{x-1}\ \underline{P}, & x = 1, 2, 3, 4, \ldots \\ 0, & o.w \end{cases}$$

Probability of failure

Probability of success

لما في x=0 ، لأنو مستحيل أحصل على success بدون ما أجرب ،
يعني x=0 إني أحصل على success بعد ولا تجربة (zero trials) ، فهاد مستحيل ، لازم أجرب لو مرة واحدة ، عشان أحصل على success .
∴ لما x=0 بنرجع كنا ال o.w .

Theorem:- The mean and the variance of (X) are:-

$$\mu_X = E\{X\} = \frac{1}{P}$$ probability of success

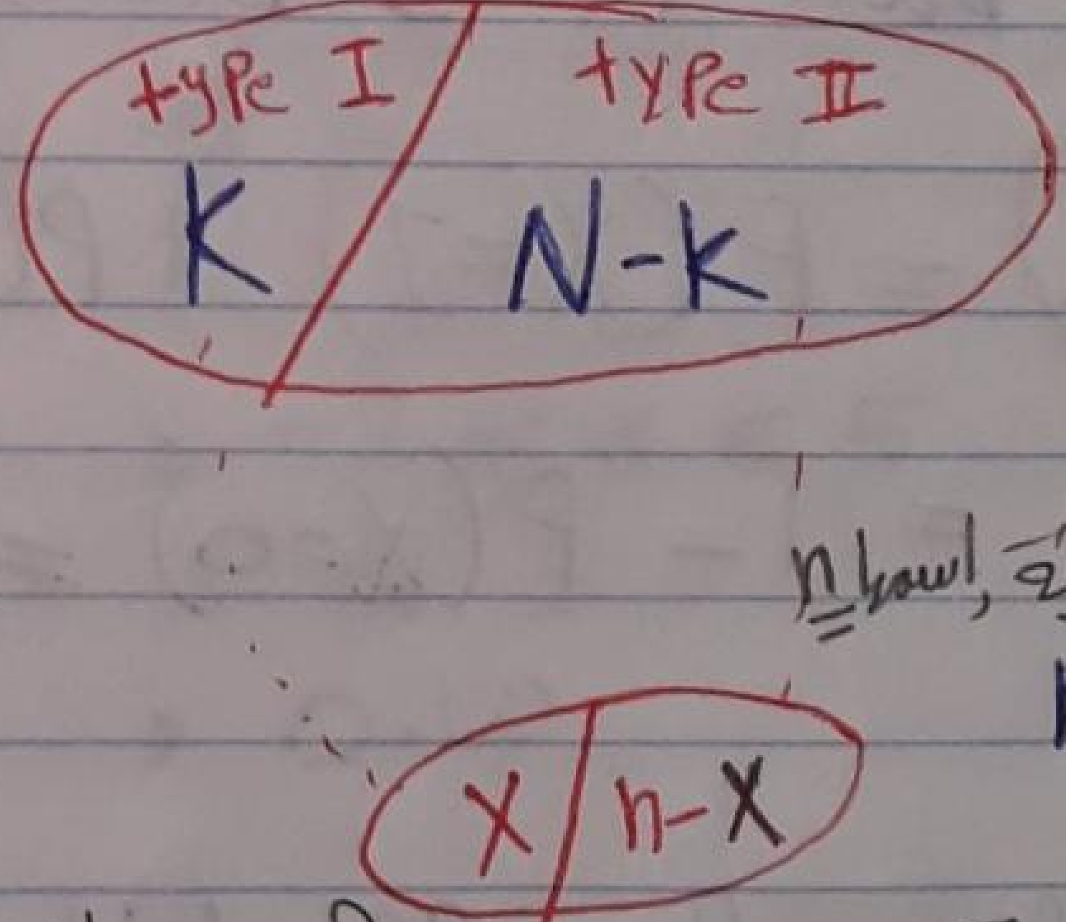
$$\sigma_X^2 = Var(X) = \frac{1-P}{P^2}$$ ← probability of failure ← " " success تربيع

ال examples بالدوسية

3 Hyper-Geometric Distribution:-

Consider the sampling without replacement of a lot of (N) items, (k) of which are of type one, and N-K of type two. The probability of obtaining (x) items in a selection of n items without replacement obeys the hyper-geometric distribution:-

* كدة type I بتكون أصغر من type II



N مجموع كل العناصر

بناخذ من N عينة, اسمها n

n

X:- number of items from group I (type I).

هاد إلكن هوال: ما وهاد Hyper geometric distribution

$$P(X=x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-k}{n-x}}{\binom{N}{n}} & , x=0,1,2,\dots \min(n,k) \\ 0 & , O.W \end{cases}$$

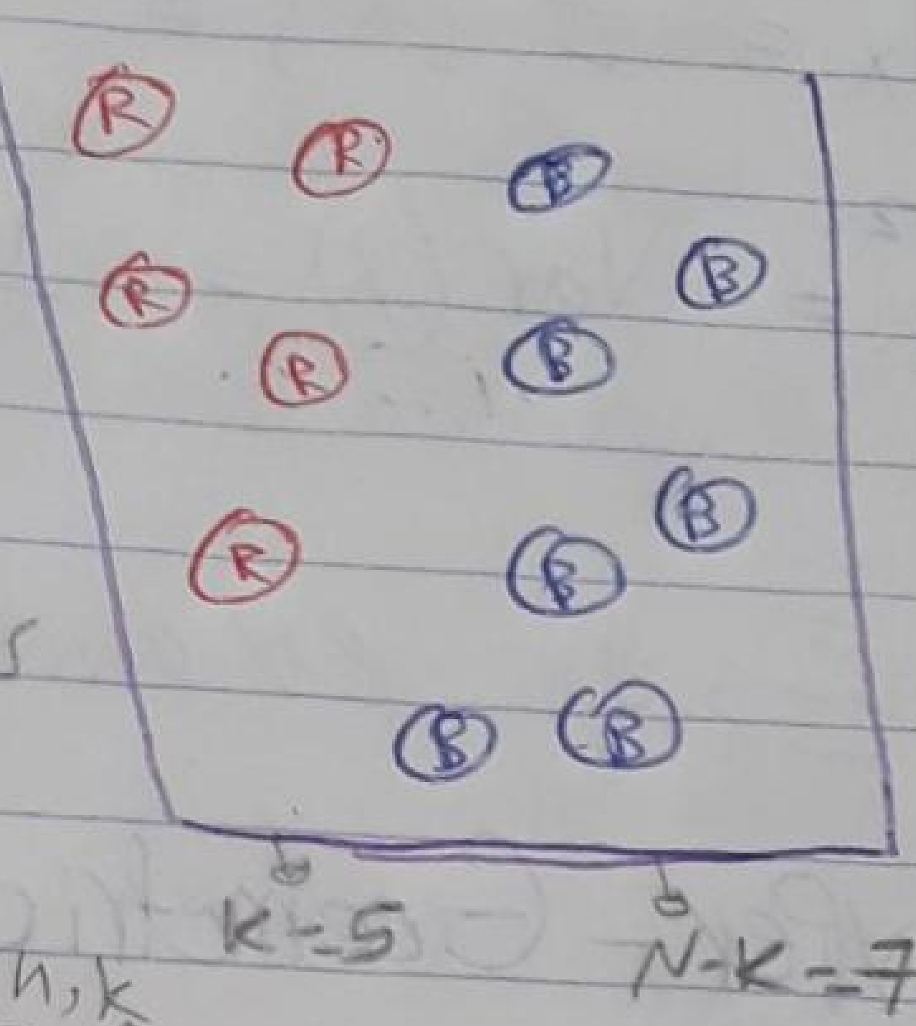
min(n,k) لما الـ minimum Value between n and k.

Scanned by TapScanner

Example: (a) what is the Probability of having two red balls in 3 selected balls?

Using the hyper-Geometric Distribution:-

$N=12$
$K=5$
$N-K=7$



X:- having two red balls in 3 selected balls

$$P(X=x)=\frac{\binom{5}{x}\binom{7}{3-x}}{\binom{12}{3}}, x=0,1,2,\dots \min\binom{n,k}{3,5}$$

$, x=0,1,2,3$

X | 3-X

$n=3$

$$P(X=2)=\frac{\binom{5}{2}\binom{7}{1}}{\binom{12}{3}}$$

لاحظ ان مجموع البسط يساوي المقام

وهيك لازم دائماً غير هيك الكل غلط .

(b) what is the Probability of selecting at least one red ball if 3 balls are to be selected.

$$P(X\geq 1)=P(X=1)+P(X=2)+P(X=3)$$

$$=1-P(X=0)=1-\frac{\binom{5}{0}\binom{7}{3}}{\binom{12}{3}}$$

(c) what is the Probability of getting at least 4 red balls if 7 balls are to be selected.

X: getting at least 4 red balls

R: $K=5$ / B: $N-K=7$

$$P(X=x)=\frac{\binom{5}{x}\binom{7}{7-x}}{\binom{12}{7}}, x=0,1,\dots \min(n,k)\ \boxed{5}$$

$$P(X\geq 4)=P(X=4)+P(X=5)$$

$\Leftarrow (P(0)+P(1)+P(2)(3))-1$

X / 7-X $n=7$

$$=\frac{\binom{5}{4}\binom{7}{3}}{\binom{12}{7}}+\frac{\binom{5}{5}\binom{7}{2}}{\binom{12}{7}}$$

Scanned by TapScanner

IV Poisson Process :- كل المعلومات تبعهم موجودة بال notes

$\boxed{\mu_x = }$, $\sigma_x^2 = $? , $\sigma_x = \sqrt{\sigma_x^2}$

- ***Poisson Process:***

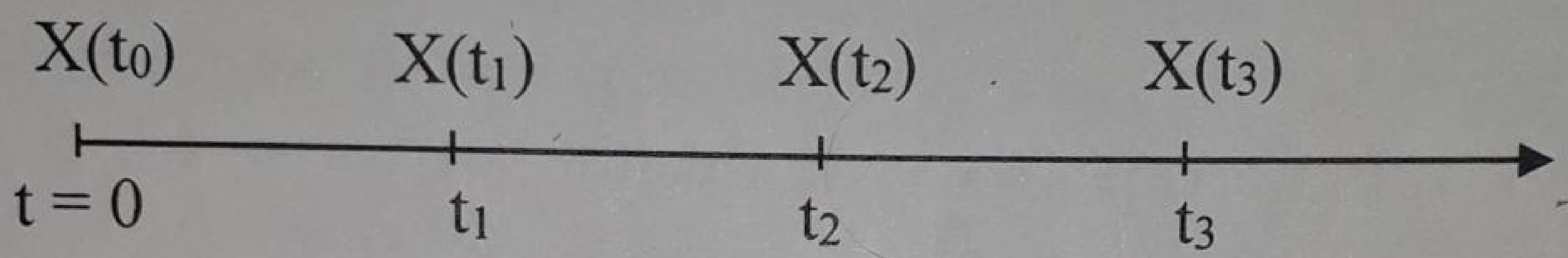
Consider a counting process in which events occur at a rate of (λ) occurrence per unit time. Let $X(t)$ be the number of occurrences recorded in the interval $(0 , t)$, we define the *Poisson process* by the following assumptions:

1- $X(0) = 0$, i.e., we begin the counting at time $t = 0$.

2- For non-overlapping time intervals $(0 , t_1)$, (t_2 , t_3), the number of occurrences $\{X(t_1) - X(0)\}$ and $\{X(t_3) - X(t_2)\}$ are independent.

3- The probability distribution of the number of occurrences in any time interval depends only on the length of that interval.

4- The probability of an occurrence in a small time interval (Δt) is approximately ($\lambda \, \Delta t$).



Using the above assumptions, one can show that the probability of exactly (x) occurrences in any time interval of length (T) follows the Poisson distribution and,

$$P(X = x) = e^{-\lambda T} \frac{(\lambda T)^x}{x!} \quad ; \quad x = 0 , 1 , 2 , 3 , \ldots\ldots$$

$P(X = x) = e^{-\lambda T} \frac{(\lambda T)}{x!}$

- ***Theorem:***

Let (b) be a fixed number and (n) any arbitrary positive integer. For each nonnegative integer (x):

$$\lim_{n\to\infty} \binom{n}{x} (p)^x (1 - p)^{n-x} = e^{-b} \frac{b^x}{x!} \quad ; \text{ where } p = b/n$$

EXAMPLE (3-21):

Messages arrive to a computer server according to a Poisson distribution with a mean rate of 10 messages/hour.

a- What is the probability that 3 messages will arrive in one hour.

b- What is the probability that 6 messages will arrive in 30 minutes.

SOLUTION:

a- $\lambda = 10$ messages/hour $\rightarrow$ $T = 1$ hour

$$P(X = x) = e^{-10\times 1} \frac{(10 \times 1)^x}{x!} = e^{-10} \frac{(10)^x}{x!} \quad ; \quad x = 0 , 1 , 2 , 3 , \ldots\ldots$$

$$P(X=3)=e^{-10}\frac{(10)^3}{3!}$$

b- $\lambda = 10$ messages/hour → T = 0.5 hour

$$P(X=x)=e^{-10\times\frac{1}{2}}\frac{(10\times\frac{1}{2})^x}{x!}=e^{-5}\frac{(5)^x}{x!} \quad ; \quad x=0,1,2,3,\ldots\ldots\ldots$$

$$P(X=6)=e^{-5}\frac{(5)^6}{6!}$$

EXAMPLE (3-22):

The number of cracks in a section of a highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.

a- What is the probability that there are no cracks in 5 miles of highway?
b- What is the probability that at least one crack requires repair in ½ miles of highway?
c- What is the probability that at least one crack in 5 miles of highway?

SOLUTION:

a- $\lambda = 2$ cracks/mile → T = 5 miles

⇒ b = λT = 2 cracks/mile · .5 miles = 2 cracks

period

المسؤول عنه

$$P(X=x)=e^{-2\times 5}\frac{(2\times 5)^x}{x!}=e^{-10}\frac{(10)^x}{x!} \quad ; \quad x=0,1,2,3,\ldots\ldots\ldots$$

لازم الوحدات (Period) يكونوا مع بعض ويشيل الإشي اللي بسأل عنه

$$P(X=0)=e^{-10}$$

b- $\lambda = 2$ cracks/mile → T = 0.5 miles

$$P(X=x)=e^{-2\times\frac{1}{2}}\frac{(2\times\frac{1}{2})^x}{x!}=e^{-1}\frac{(1)^x}{x!}=\frac{e^{-1}}{x!} \quad ; \quad x=0,1,2,3,\ldots\ldots\ldots$$

$$P(X\geq 1)=\sum_{x=1}^{\infty}\frac{e^{-1}}{x!}=[1-P(X=0)]=1-e^{-1}$$

c- $\lambda = 2$ cracks/mile → T = 5 miles ✓

$$P(X=x)=e^{-2\times 5}\frac{(2\times 5)^x}{x!}=e^{-10}\frac{(10)^x}{x!} \quad ; \quad x=0,1,2,3,\ldots\ldots\ldots$$

$$P(X\geq 1)=\sum_{x=1}^{\infty}\frac{e^{-10}(10)^x}{x!}=[1-P(X=0)]=1-e^{-10}$$

EXAMPLE (3-23):

Given 1000 transmitted bits, find the probability that exactly 10 will be in error. Assume that the bit error probability is $\frac{1}{365}$.

SOLUTION:

X: random variable representing number of bits in error.

Exact solution:

$P(\text{bit error})=\frac{1}{365}$; Number of trials (n) = 1000

Required number of bits in error (k) = 10

$$P(X=10)=\binom{n}{k}(p)^k(1-p)^{n-k}=\binom{1000}{10}\left(\frac{1}{365}\right)^{10}\left(\frac{364}{365}\right)^{990}$$

Approximate solution:

$$P(X = x) = e^{-b}\frac{b^x}{x!} \quad ; \; b = n\,p = 1000 \times \frac{1}{365} = \frac{1000}{365}$$

$$P(X = 10) = e^{-b}\frac{b^{10}}{10\,!}$$

Exercise:

Perform the computation and compare the difference

Scanned by TapScanner

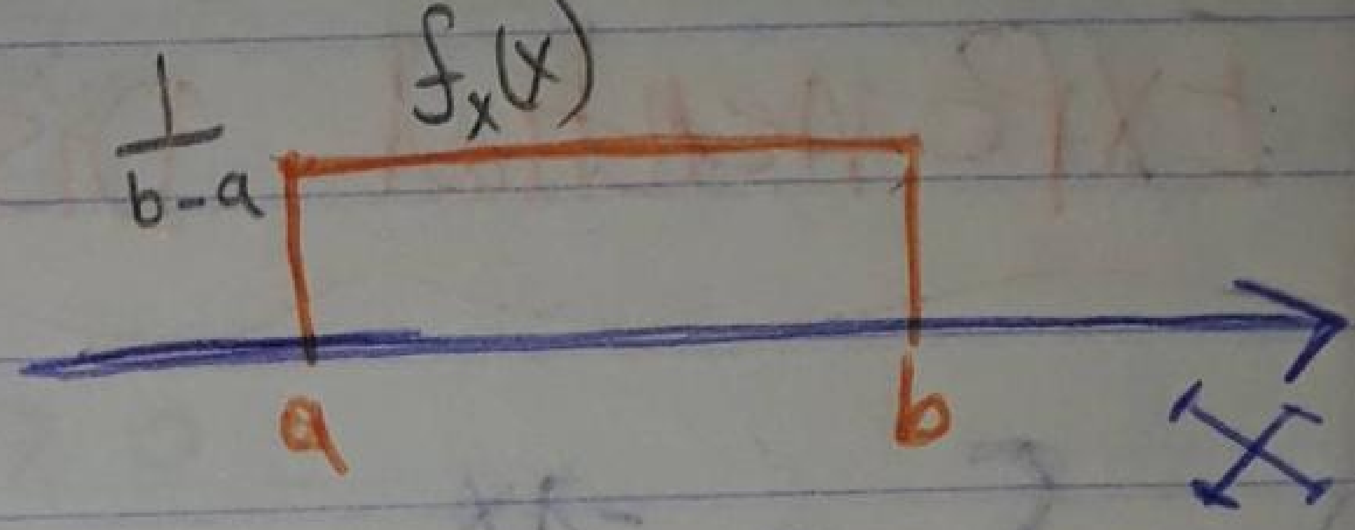
Common continuous Random Variables

mass density

لما أخذنا continuous الـ X's لهم pdf وليس PMF

I Uniform Distribution

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & o.w \end{cases}$$



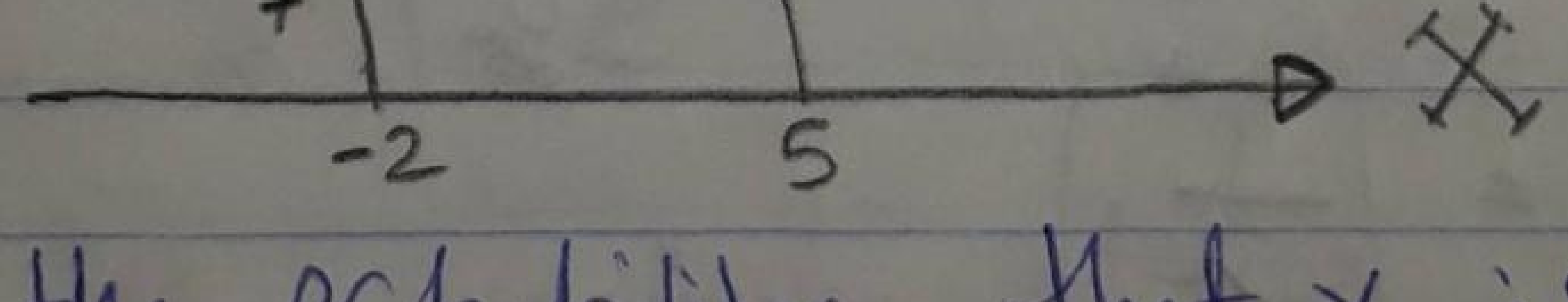
$\mu_X = \frac{a+b}{2}$ mean إثبات $\Rightarrow \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_a^b \frac{x}{b-a}\, dx = \frac{a+b}{2}$

$\sigma_X^2 = \frac{(b-a)^2}{12}$

Example :- let X be a R.V that follows uniform distribution in the interval $[-2, 5]$.

① Write and plot the pdf of X.

$$f_X(x) = \begin{cases} \frac{1}{b-a} = \frac{1}{5--2} = \frac{1}{7}, & -2 \le x \le 5 \\ 0, & o.w \end{cases}$$



② What is the probability that X is less than zero?

$P(X<0) = ? \Rightarrow P(X<0) = \int_{-\infty}^{0} f_X(x)\, dx = \int_{-2}^{0} \frac{1}{7}\, dx = \frac{2}{7}$

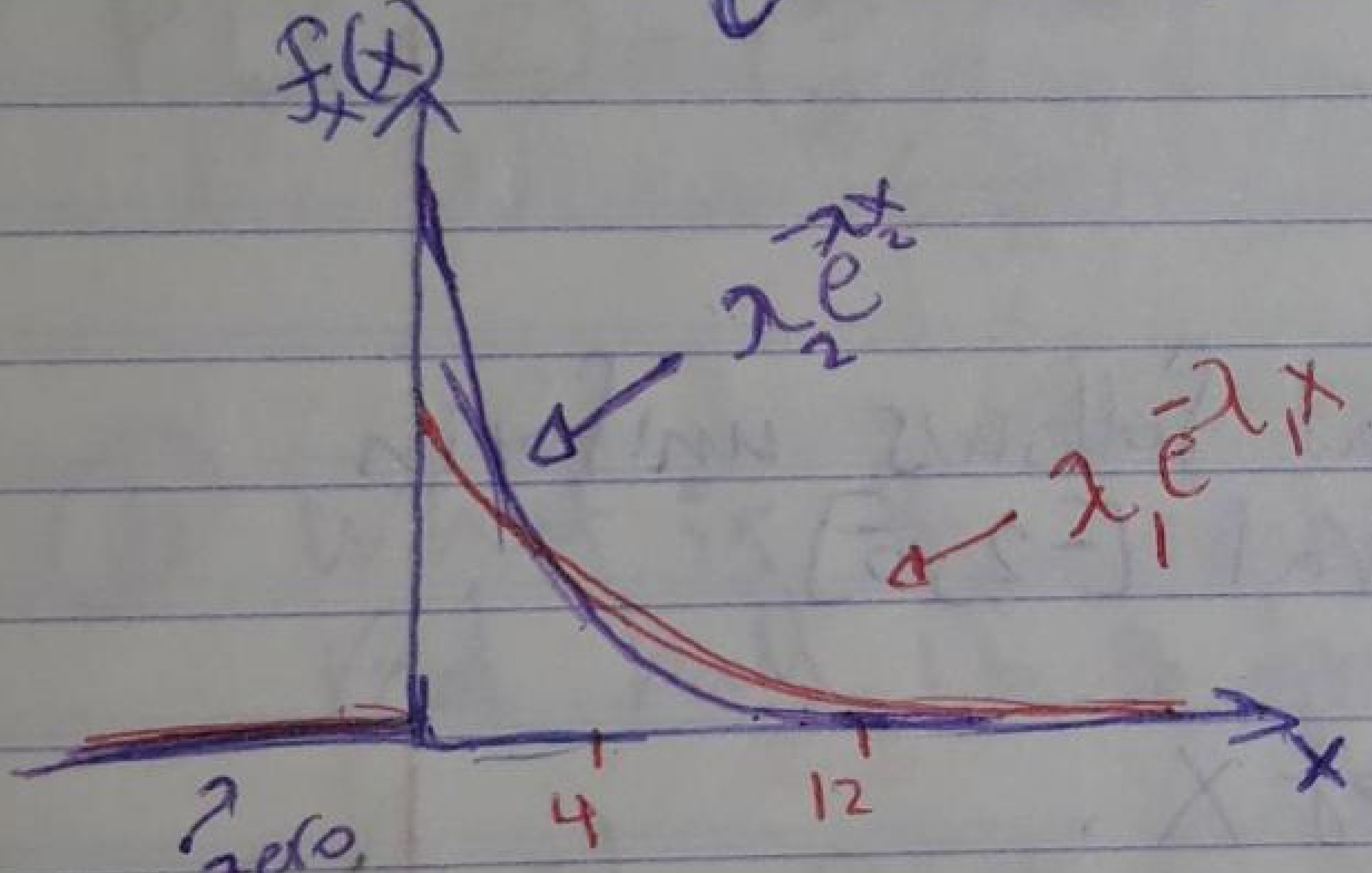
③ Determine the mean and variance of X ?

$$\mu_x = \frac{a+b}{2} = \frac{-2+5}{2} = 1.5$$

$$\sigma_x^2 = \frac{(b-a)^2}{12} = \frac{(5+2)^2}{12} = \frac{49}{12}$$

2 Exponential Distribution

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{o.w} \end{cases}$$



$\lambda_2 > \lambda_1$

المنحنى تبع λ_2 بنقص
بشكل أسرع من المنحنى تبع λ_1،
وهاد بيعني إذا في machine، waiting، bus، بنستنى على الطابعة
فدقائق الانتظار تبعت λ_2 تكون أقل من دقائق λ_1، عشان عملية الأداء
رح تكون أسرع في λ_2 من λ_1.

$\mu_x = \frac{1}{\lambda}$ إثباتها $\Rightarrow$ $\mu_x = \int_{-\infty}^{\infty} x f_x(x)\, dx = \int_{-\infty}^{\infty} x \lambda e^{-\lambda x} dx$

تكامل بالأجزاء بيرها $\longrightarrow$ ..... = $\boxed{\frac{1}{\lambda}}$

$\sigma_x^2 = \frac{1}{\lambda^2}$ إثباتها $\Rightarrow$ $\sigma_x^2 = E\{x^2\} - (\mu_x)^2$

$\longrightarrow$

Scanned by TapScanner

$$E\{x^2\} = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx = \int_{0}^{\infty} x^2 f_x(x)\,dx \;\ldots\ldots = \frac{1}{\lambda^2}$$

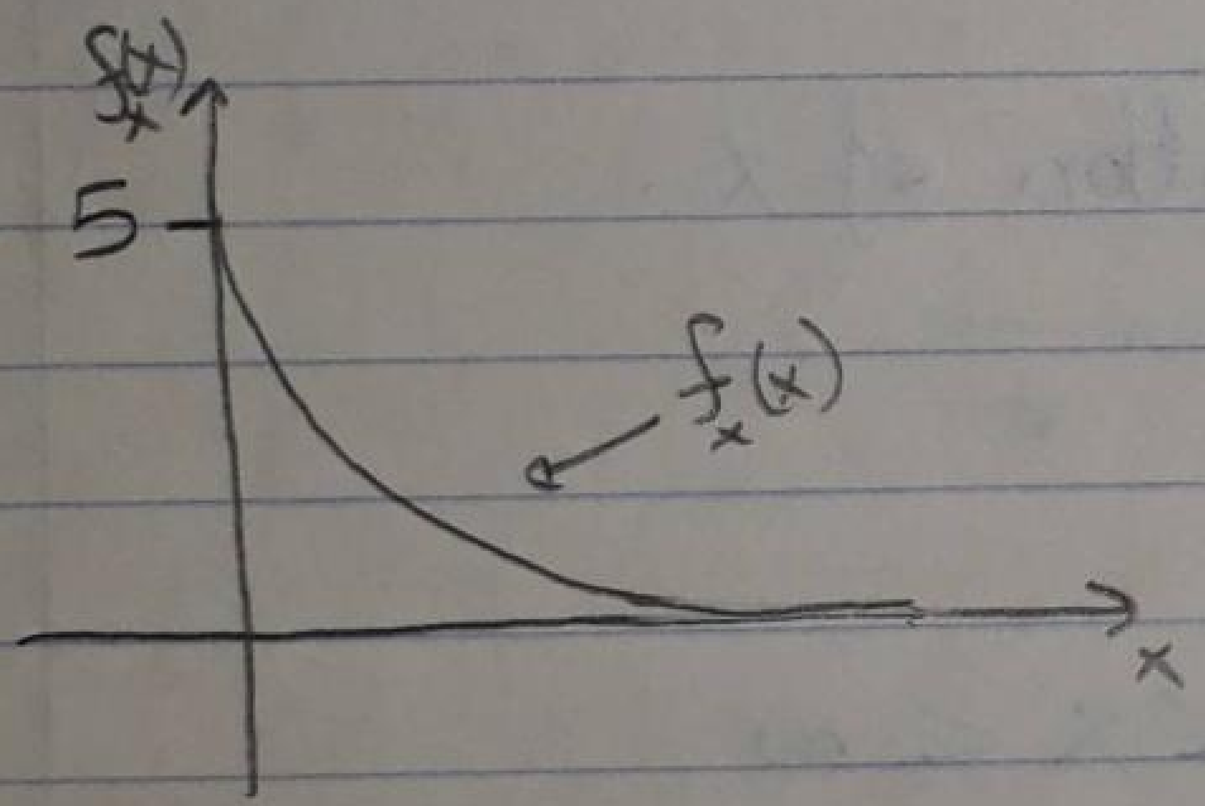
Example :- let X be an exponential R.V with a mean of 0.2

ⓐ write and plot the Pdf of X.

$$\mu_X = \frac{1}{\lambda} = 0.2 \Rightarrow \lambda = \frac{1}{\mu_X} = \frac{1}{0.2} = 5$$

$$f_x(x) = \begin{cases} 5e^{-5x}, & x \geq 0 \\ 0, & \text{o.w} \end{cases}$$

Pdf as it is continuous



ⓑ $P(X \leq 2) = ?$

$$P(X \leq 2) = \int_{-\infty}^{2} f_x(x)\,dx = \int_{0}^{2} 5e^{-5x}\,dx = \left.\frac{5e^{-5x}}{-5}\right|_0^2$$

$$= -e^{-10} + 1 = 1 - e^{-10}.$$

ⓒ Determine the variance of X?

$$\sigma_x^2 = \frac{1}{\lambda^2} = \frac{1}{5^2} = \frac{1}{25} = 0.04$$

Scanned by TapScanner

3 Gaussian (Normal) Distribution

A random Variable (X) with Pdf :-

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} \quad , \quad -\infty < X < \infty$$

$$\mu_X = E\{X\} \quad , \quad Var(X) = \sigma_X^2$$

في رسمات مع ملاحظات عليهم موجودين بالدوسيّة عن ال Gaussian .

Example :- let X be a Normal R.V with zero mean and unity variance ، $[\mu_X = 0 , \sigma_X^2 = 1]$

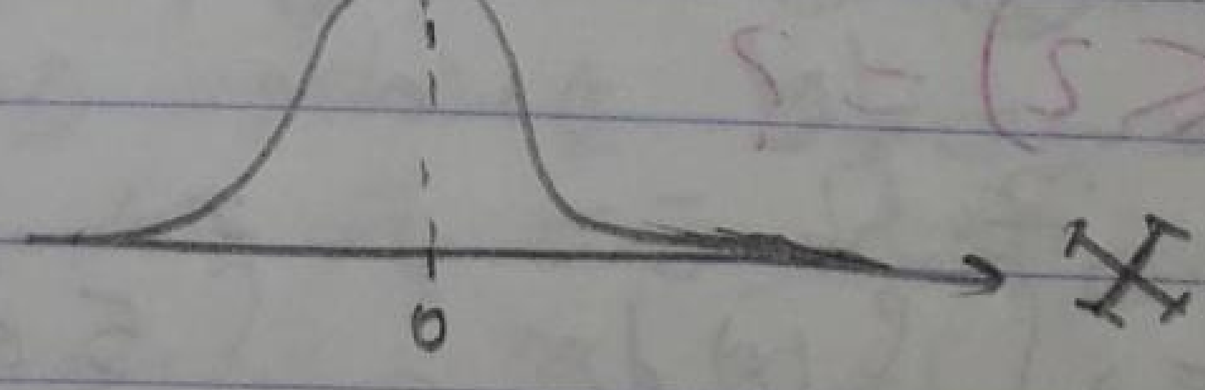
$\underbrace{\text{unity}}_{\text{وحدة}\ 1 = \sigma}$

(a) write and Plot the distribution of x.

Normal → Gaussian →

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad , \quad -\infty < X < \infty$$



(b) $P(X<0) = ?$

$$P(X\leq 0) = \int_{-\infty}^{\boxed{0}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

→ ما بنحلها عن طريق التكامل
لانه اجزاء يعني صعب وما رح نعرف
فبنروح على الجدول الخاص بال Standard Normal (Gaussian) distribution
الي ال $\mu_X = 0$ و ال $\sigma_X^2 = Variance = 1$

$$= \phi(0) = 0.5$$

وهو منطقي عشان ال 0
تمثل ال mean يعني كأننا بنقسم
ال curve الى قسمين متساويين

(c) $P(X \leq 1.12) = ??$ $\rightarrow$ Gaussian ✓, $\mu_X = 0$ ✓, $\sigma_X^2 = 1$ ✓ $\Big\}$ $\therefore$ Apply on the values of Standard Normal distribution Table.

$\therefore P(X \leq 1.12) = \phi(1.12) = 0.8686$

(d) $P(X \geq 3.12) = ??$

ما عطينا 3.199999 عشان بالجدول فش قيمة بهاي الدقة، بس موجود أقرب منزلتين عشريتين، فخلص باخذ الـ 3.12.

$P(X \geq 3.12) = 1 - P(X < 3.12) = 1 - \phi(3.12)$

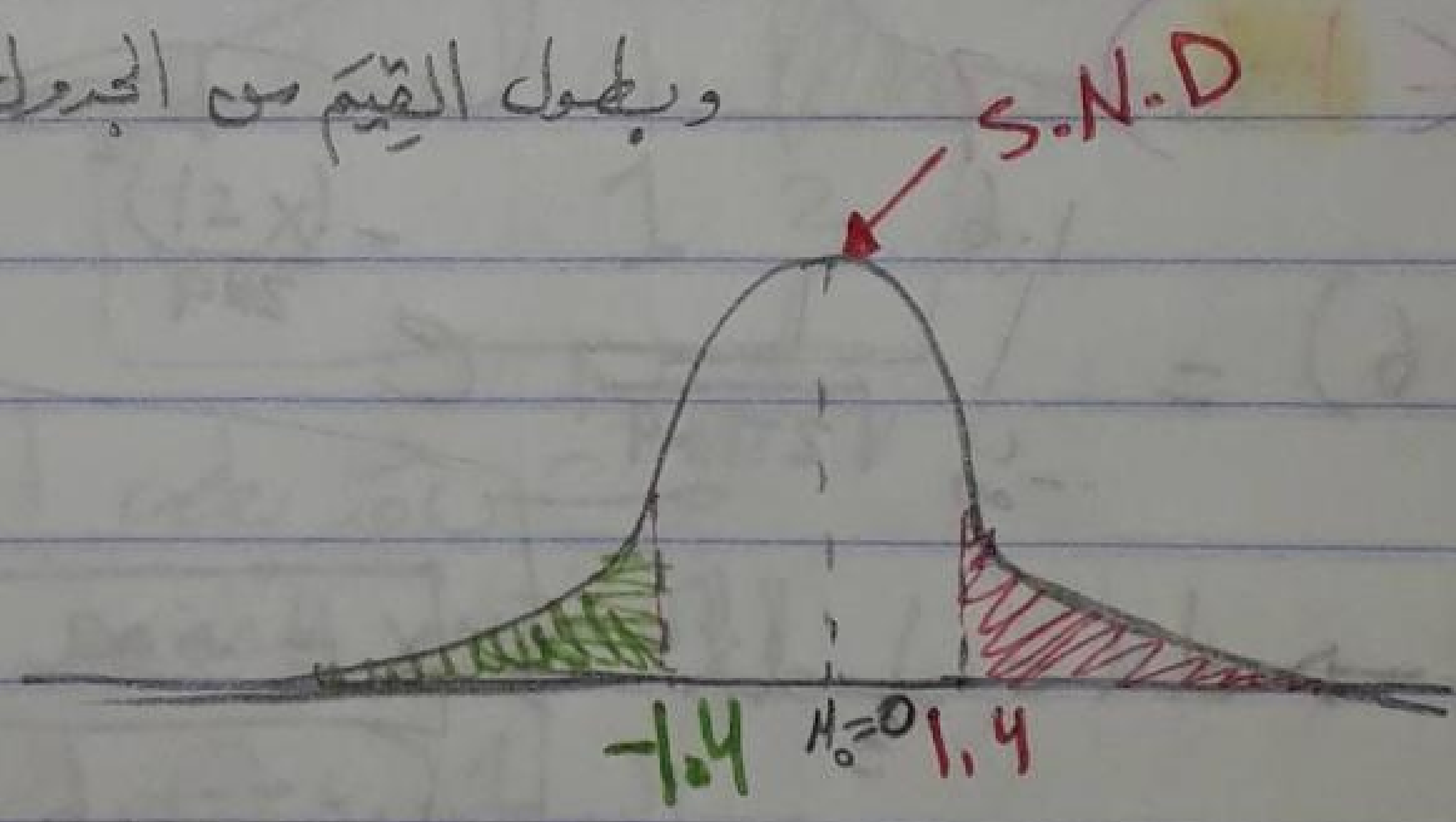
$= 1 - 0.9991 = 0.0009$

(e) $P(0.5 \leq X \leq 1.7) = ?$

$P(0.5 \leq X \leq 1.7) = P(X \leq 1.7) - P(X < 0.5)$

$= \phi(1.7) - P(0.5) \rightarrow$ وبطول القيم من الجدول

(f) $P(X \leq -1.4) = ??$



$P(X \leq -1.4) = P(X \geq 1.4)$

$= 1 - P(X < 1.4)$

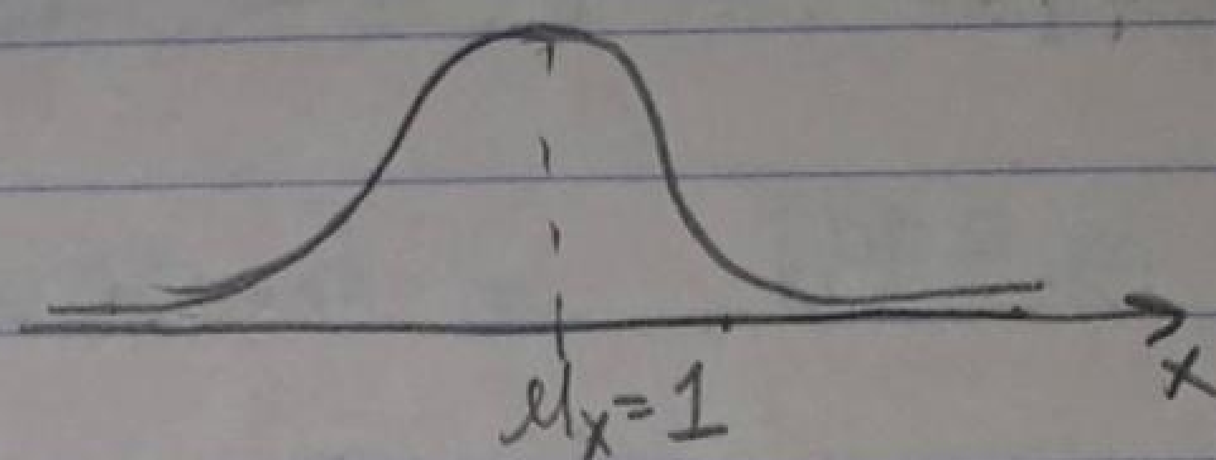
$\phi(-1.4) = 1 - \phi(1.4)$

$$\boxed{\phi(-a) = 1 - \phi(a)}$$

Example:- Let X be a Normal R.V with $\mu_x = 1$, $\sigma_x^2 = 9$.

ⓐ write and Plot the pdf of X.

$$f_x(x) = \frac{1}{\sqrt{2\pi * 9}} e^{\frac{-(x-1)^2}{2*9}} \quad , \quad -\infty < x < \infty$$



ⓑ $P(X \leq 1) = ?$

$$P(X \leq \underset{\mu_x}{1}) = 0.5$$

ⓒ $P(X \leq 1.6) = ?$

$$P(X \leq 1.6) = \int_{-\infty}^{1.6} \frac{1}{\sqrt{2\pi * 9}} e^{\frac{-(x-1)^2}{2*9}} dx = \int_{-\infty}^{1.6} \frac{1}{\sqrt{2\pi 9}} e^{-\frac{1}{2}\left(\frac{x-1}{3}\right)^2} dx$$

$$u = \frac{x-1}{3} \rightarrow du = \frac{1}{3} dx \quad , \quad \boxed{x = -\infty \rightarrow u = -\infty} \quad \boxed{x = 1.6 \rightarrow u = \frac{1.6-1}{3}}$$

$$\therefore P(X \leq 1.6) = \int_{-\infty}^{\frac{1.6-1}{3}} \frac{1}{\sqrt{2\pi * \cancel{9}}} e^{-\frac{1}{2}u^2} \frac{\cancel{3}\, du}{dx}$$

$$= \underbrace{\int_{-\infty}^{\frac{1.6-1}{3}} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du}_{} = \phi\left(\frac{1.6 - \overset{\text{mean}}{1}}{\underset{\sqrt{Var(x)} = \sqrt{\sigma_x^2} = \sigma}{3}}\right) = \phi\left(\frac{0.6}{3}\right) = \phi(0.2)$$

جذر الـ Variance =

صار مكتوب على الصيغة
تبع الـ Standard Normal
Distribution
اللي بالصفحة القبل
b فرع 6

بقدر أستخدم هاي الصيغة لحل
المسائل اللي الـ mean تبعها $\neq 0$
أو الـ $\sigma_x^2 \neq 1$.

(d) $P(X \le 4) = ??$

$$P(X \le 4) = \phi\left(\frac{4 - \mu_x}{\sqrt{\sigma_x^2}}\right) = \phi\left(\frac{4-1}{\sqrt{9}}\right) = \phi(1)$$

Normal

$\mu_x = 1$

$\sigma_x^2 = 9$

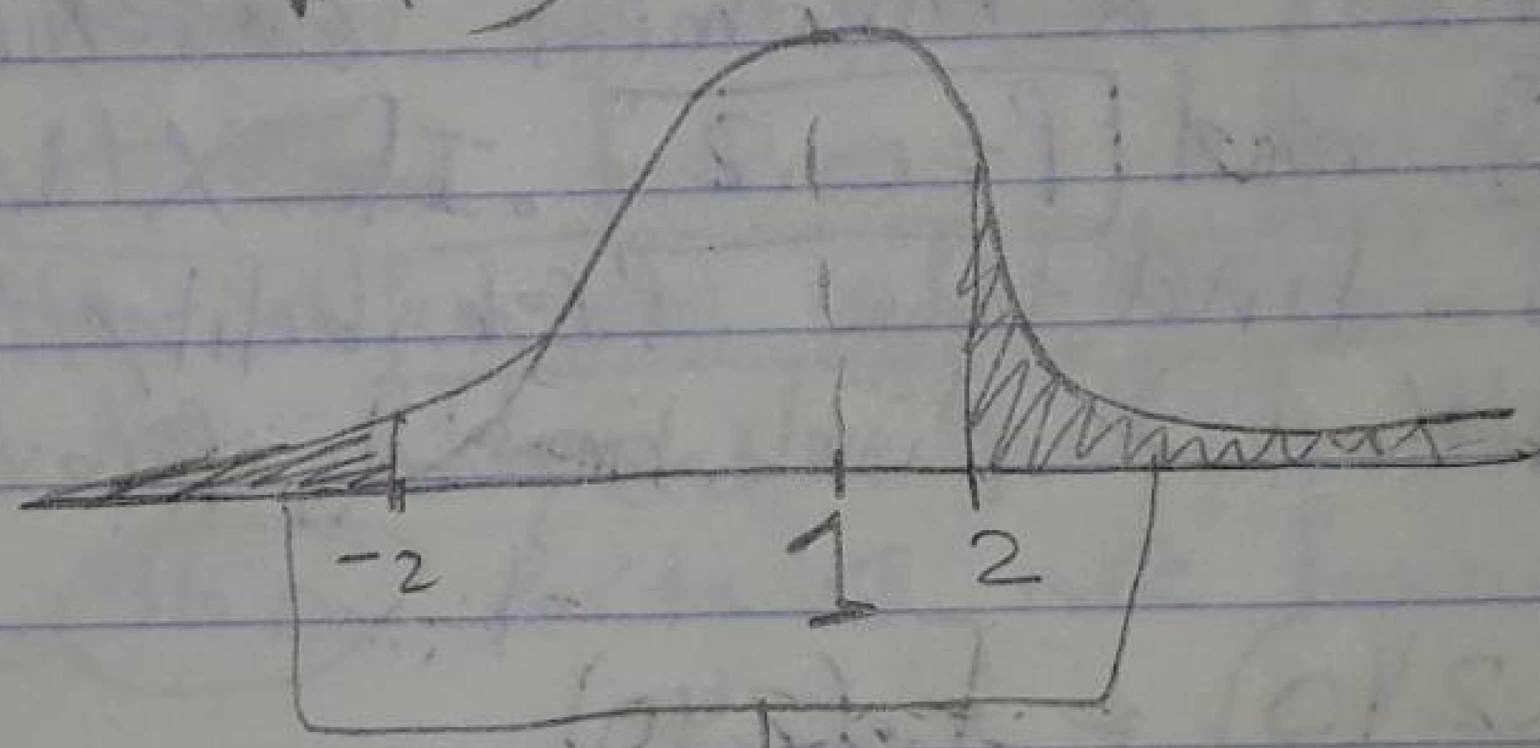
— قانون جدول Z

جذر ال Variance = ال standard deviation

(e) $P(X \le -2) = ?$

$$\phi(X \le -2) = \phi\left(\frac{-2-1}{\sqrt{9}}\right) = \phi(-1)$$

$$= 1 - \phi(1)$$



مش قد بعض ابدا لانه
الي بيقسم المنحنى نص النص هو
ال 1 مش الصفر ، عشان هيك
بنستخدم الطريقة أعلاه عشان نجيب
القيمة المكافئة للسالب المعطى لما تقسم
المنحنى لقيمة أخرى غير الصفر.

(f) $P(-2.3 \le X \le 5.3) = ??$

$$P(-2.3 \le x \le 5.3) = P(X \le 5.3) - P(X > -2.3)$$

$$= \phi\left(\frac{5.3-1}{\sqrt{9}}\right) - \phi\left(\frac{-2.3-1}{\sqrt{9}}\right) = \phi\left(\frac{4.3}{3}\right) - \phi\left(\frac{-3.3}{3}\right)$$

$$= \phi\left(\frac{4.3}{3}\right) - [1 - \phi(1.1)]$$

$$= \phi(1.43) - [1 - \phi(1.1)].$$

1.43 بالتقريب نقربها.

cdf يعني البروبابيليتي للإكس أقل أو يساوي 2.3-

9 $F_X(-2.3) = ??$

$$F_X(-2.3) = P(X \leq -2.3)$$
$$= \phi\left(\frac{-2.3-1}{\sqrt{9}}\right) = \phi\left(\frac{-3.3}{3}\right) = \phi(-1.1)$$
$$= 1 - \phi(1.1)$$

Normal (continuous) Approximation of the Binomial (discrete) and Possion (discrete) Distribution

Theorem :- De-Moiver-laplace :-

ex:- Consider a binomial experiment with $\boxed{n = 1000}$ and $\boxed{P = 0.2}$. If X is the number of successes. find the Probability that $X \leq 240$.

بدي أعمل approximation للbinomial باستخدام ال Normal (Gaussian).

$$\therefore P(X \leq 240) = F_X(240)$$

$$P(X = x) = \binom{1000}{x} (0.2)^x (0.8)^{1000-x}, \quad x = 0, 1, 2, \dots, 1000$$

Exact solution :- $P(X \leq 240) = \sum_{x=0}^{240} \binom{1000}{x} (0.2)^x (1-0.2)^{1000-x}$

وهاد الإشي كتير صعب يعني بدي أقعد أحسب الأرقام من صفر لحد 240، بعدين أجمعهم، وهاد الإشي جداً صعب، عشان هيك بنعمل approximation

باستخدام ال Normal، يعني نظرية De-Moiver :-

Applying the De Moiver-Laplace theorem :-

$$P(X \leq 240) = \phi\left(\frac{240 - \mu_X}{\sqrt{\sigma_X^2}}\right) \Rightarrow \begin{aligned} \mu_X &= n\,P(S) = 1000 * 0.2 \\ &= 200 \\ \sigma_X^2 &= n\,P(S)\,[1 - P(S)] \\ &= 1000 * 0.2 * 0.8 \\ &= 160 \end{aligned}$$

فبنفرض إنو ال mean تبع ال binomial هو نفسو ال mean تبع ال Gaussian، نفس الإشي بالنسبة لل σ_X^2 (Variance).

$$P(X \leq 240) = \phi\left(\frac{240-200}{\sqrt{160}}\right) = \phi(3.16) = 0.9992$$

The therem gives better results when

$\underbrace{np}_{mean} > 5$ and $\underbrace{npq}_{Variance} > 5$

Normal Approximation for Poisson Distribution

Ex Assume the number of asbestos particles in a $\boxed{cm^3}$ of dust ~~from~~ follow a $\boxed{\text{Poisson}}$ $\boxed{\text{distribution}}$ with a $\boxed{\text{mean of }1000}$. If a cm^3 of dust is analyzed. what is the Probability that less than 950 particles are found in $1\ cm^3$?

In Possion Distribution:- $\mu_x = \sigma_x^2 = b$

$$b = \lambda T = 1000 \frac{\text{Particles}}{\cancel{cm^3}} * 1\ \cancel{cm^3}$$

$$= 1000 \text{ Particles.}$$

$$\therefore P(X=x) = e^{-b}\frac{b^x}{x!} = e^{-1000}\frac{(1000)^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Exact Solution:- $P(X \leq 950) = \sum_{x=0}^{950} e^{-1000}\frac{(1000)^x}{x!}$

Approximation:- $P(X \leq 950) = \phi\left(\frac{950-\mu_x}{\sqrt{\sigma_x^2}}\right)$

$$= \phi\left(\frac{950-1000}{\sqrt{1000}}\right) = \phi\left(\frac{-50}{\sqrt{1000}}\right) = \phi(-1.58)$$

$$= 1 - \phi(1.58)$$

$$= 0.057$$

Transformation of Random Variables

1 Discrete case:-

Ex let X be a binomial Random variable with Parameters (n=3) and P=0.75. ~~let y=g(x)~~
let $y=g(x)=2x+3$ / $P(Y=y)=P(X=x)$.

$$P(X=x)=\begin{cases}\binom{n}{x}(0.75)^x(0.25)^{3-x}, & x=0,1,2,3\\ 0 & ,o.w\end{cases}$$

x	P(X=x)	y = 2x + 3	P(Y=y)
0	$\frac{1}{64}$	$2(0)+3 = 3$	1/64
1	$\frac{9}{64}$	$2(1)+3 = 5$	9/64
2	$\frac{27}{64}$	$2\times2+3 = 7$	27/64
3	$\frac{27}{64}$	$2\times3+3 = 9$	27/64

2 Continuous Case:-

Let $y=g(x)$ be a monotonically increasing or decreasing function of x. ~~$f_y(y)=f_x(x)$~~ $f_y(y)=\frac{f_x(x)}{\left|\frac{dy}{dx}\right|}$ ⇒ $x=\pm\sqrt{y}$

Ex

X	P(X=x)	y=g(x)=X^2
-3	1/6	$(-3)^2=9$
-2	1/6	$(-2)^2=4$
-1	1/6	$(-1)^2=1$
0	1/6	
1	1/6	
2	1/6	

P(y=y)

$P(Y=9)=P(x=\pm\sqrt{9})=P(x=-3)+P(x=3)=\frac{1}{6}$ ($\frac{1}{6}$, 0)

$P(Y=4)=P(X=\pm\sqrt{4})=P(X=-2)+P(X=2)=\frac{2}{6}$

$P(Y=1)=P(X=1)+P(X=-1)=\frac{2}{6}$

$P(Y=0)=P(X=0)=1/6$

$$\therefore P(Y=y)=\begin{cases}1/6 & , y=9\\ 2/6 & , y=4\\ 1/6 & , y=0\\ 0 & ,o.w\end{cases}$$

*$y=x^2 \Rightarrow x=\pm\sqrt{y}$

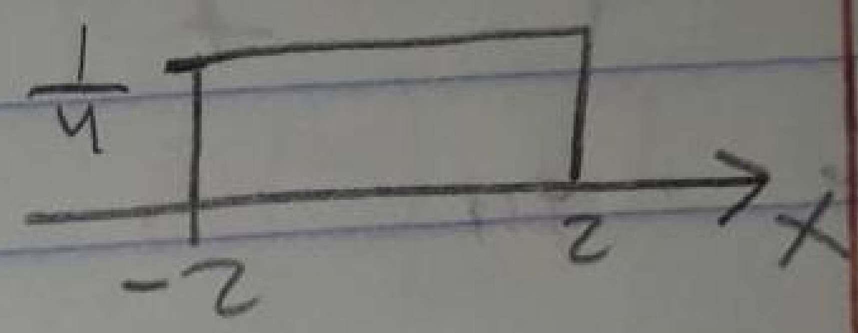
$\therefore P(y=9)=P(X=\sqrt{y})+P(X=-\sqrt{y})=P(X=3)+P(X=-3)$
$= 0 + 1/6 = 1/6$

بتاعتنا مشاكلها كتير

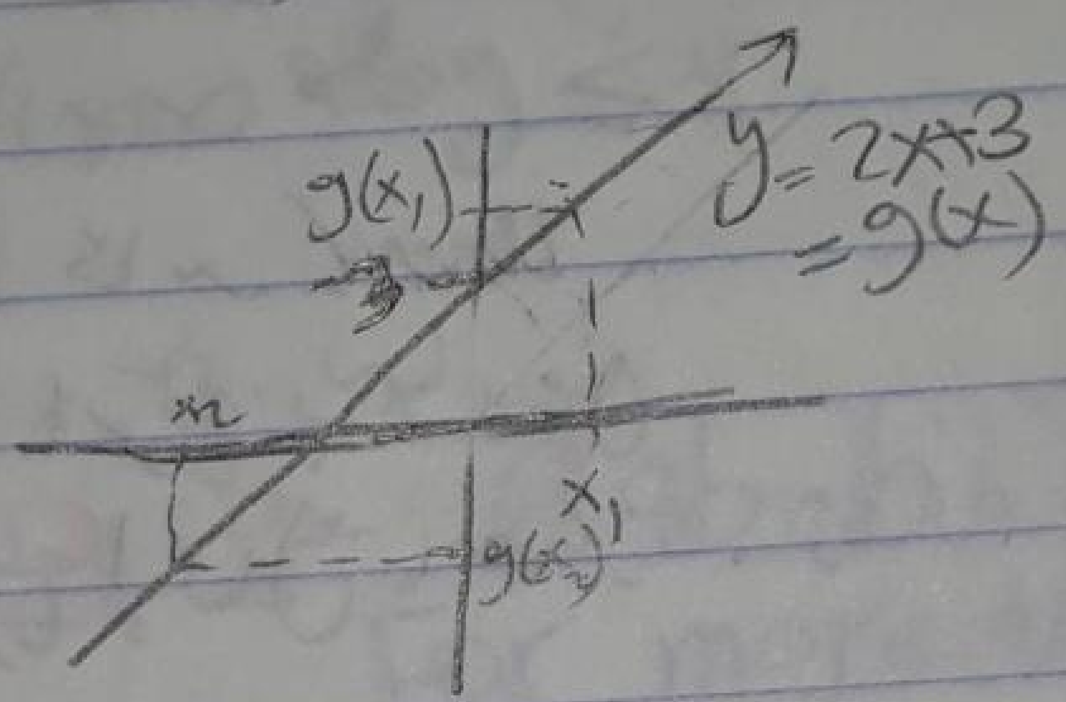
Example let X be a R.V with a uniform distribution over the interval $[-2,2]$, $Y = 2X+3$. Determine the pdf of Y.

① $f_X(x) = \begin{cases} \frac{1}{b-a} & , -2 \le x \le 2 \\ 0 & , o.w \end{cases}$

$= \begin{cases} \frac{1}{4} & , -2 \le x \le 2 \\ 0 & , o.w \end{cases}$



② $y = 2x+3$



③ يكتب x بدلالة y.

$\therefore x = \frac{y-3}{2} = g^{-1}(y)$

4 بطلع مشتقة المعكوس $\longrightarrow \frac{dy}{dx} = \frac{d(2x+3)}{dx} = 2$

5 $f_Y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x=\frac{y-3}{2}}$ ⟸ القانون اللي كتبته بالصفحة اللي قبل بالأزهري.

I $x < -2 \longrightarrow$

$f_Y(y) = 0$

$2(x) < 2(-2)$
$2x < -4$
$+3 \quad +3$
$2x+3 < -1$

حتى أوصل لقيمة واي المكافئة لـ x<-2

$\therefore x < -2 \rightarrow y < -1$

$f_Y(y) = 0$

II $-2 \le x \le 2 \longrightarrow -1 \le y \le 7$

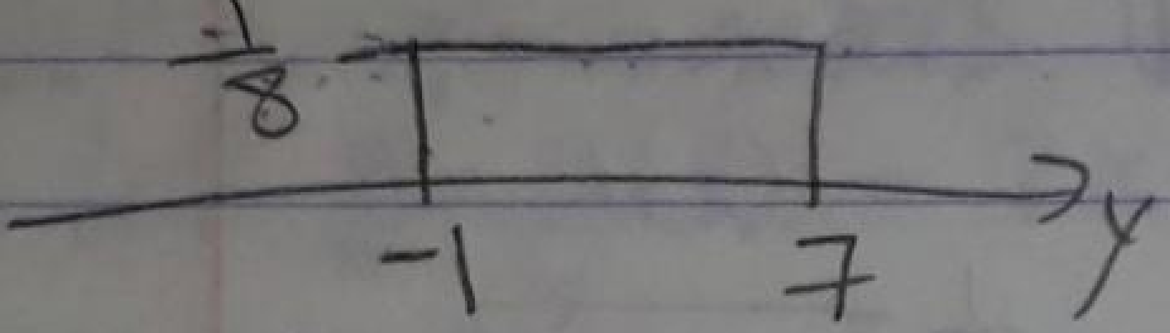
$f_Y(y) = \frac{\frac{1}{4}}{|2|} = \frac{1}{8}$

يتبع ⟹

III $2 < x \longrightarrow 7 < y$

$f_Y(y) = 0$

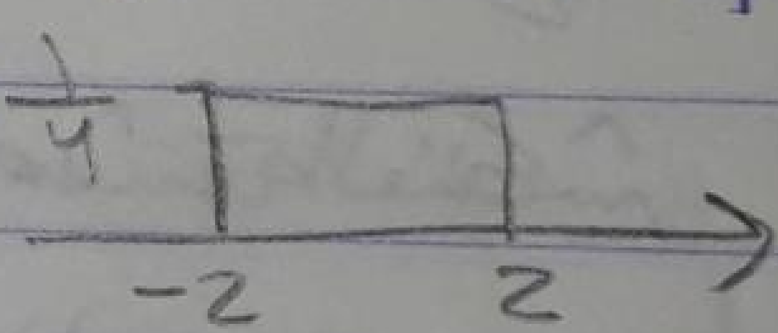
$\therefore \; f_Y(y) = \begin{cases} \frac{1}{8}, & -1 \le y \le 7 \\ 0, & o.w \end{cases}$



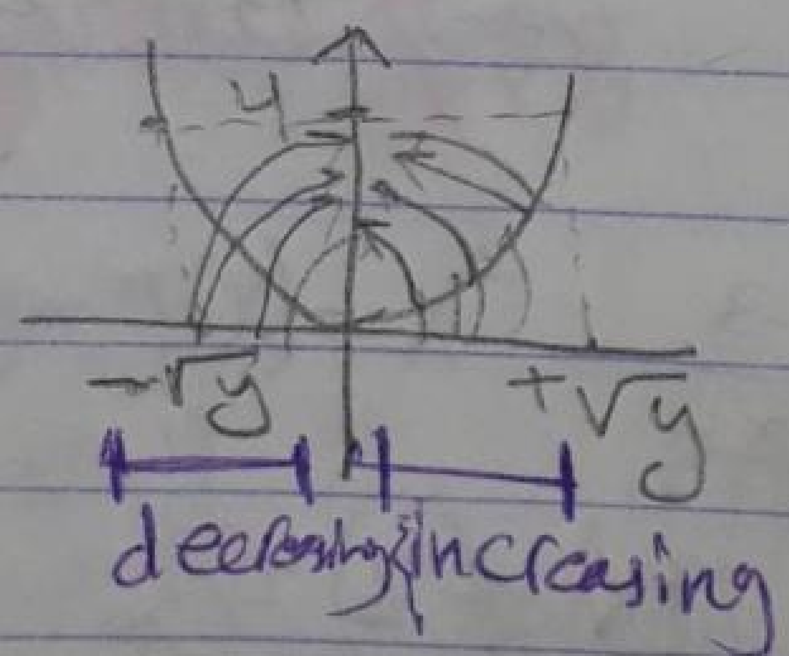
Linear transformation – (للـ Uniform)
- بحافظ على نوع الـ distribution
هون كان constant dip
constant او إذا كان linear
نفس الشيء اتربيع بـ x بيعو
تربيع بعد التحويل إلى ما وصفنا.

$\boxed{Ex}$ let X be a uniform R.V over the interval $[-2, 2]$. $Y = X^2$. Determine the pdf of Y?

① $f_X(x) = \begin{cases} \frac{1}{4}, & -2 \le x \le 2 \\ 0, & o.w \end{cases}$



② $y = g(x) = x^2$

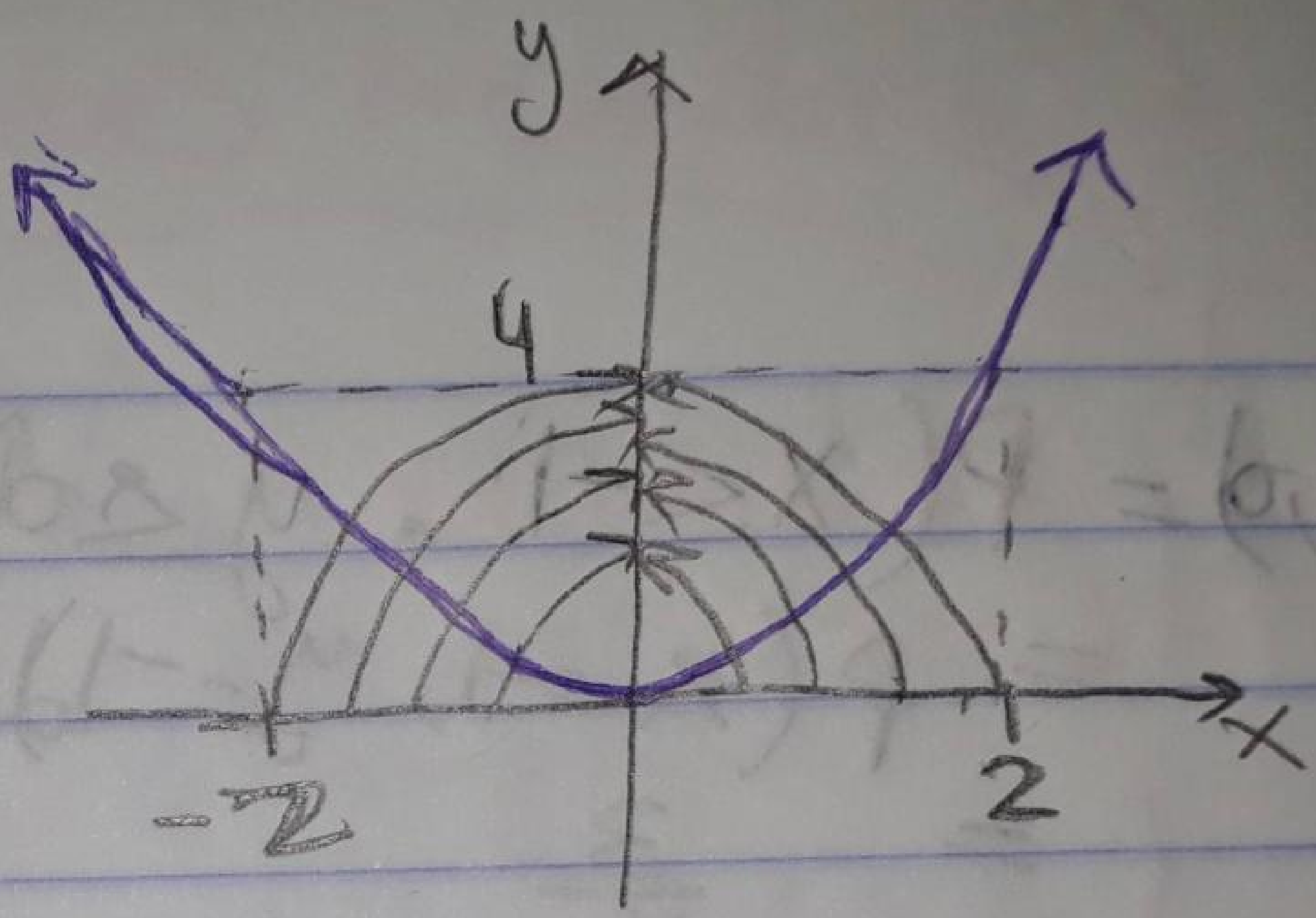


③ $x = +\sqrt{y}$ and $-\sqrt{y}$

④ $\frac{dy}{dx} = 2x$

⑤ $f_Y(y) = \left. \frac{f_X(x)}{\left|\frac{dy}{dx}\right|} \right|_{-\sqrt{y}} + \left. \frac{f_X(x)}{\left|\frac{dy}{dx}\right|} \right|_{x = +\sqrt{y}}$

يتبع



(I) $x < -2$ and $x > 2$

$\rightarrow y > 4$

$\therefore f_y(y) = 0$

(II) $-2 \leq x \leq 2 \Rightarrow 0 < y \leq 4$

الصفر مش included عشـــان ما بدنا نعرفها

$$f_y(y) = \left.\frac{\frac{1}{4}}{|2x|}\right|_{x=-\sqrt{y}} + \left.\frac{\frac{1}{4}}{|2x|}\right|_{x=\sqrt{y}} = \frac{1}{8\sqrt{y}} + \frac{1}{8\sqrt{y}} = \frac{1}{4\sqrt{y}}$$

$$\therefore f_y(y) = \begin{cases} \frac{1}{4\sqrt{y}} & , 0 < y \leq 4 \\ 0 & , \text{o.w} \end{cases}$$

Scanned by TapScanner